Topic X2 Vectors and induction (Pre-TT A) [48]

1.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{A}^n$. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

(Total 8 marks)

2. Part (i) only

Two lines have equations

$$\frac{x-k}{2} = \frac{y+1}{-5} = \frac{z-1}{-3} \quad \text{and} \quad \frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2},$$

where k is a constant.

(i) Show that, for all values of k, the lines intersect, and find their point of intersection in terms of k. [6]

(ii) For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$. [4]

(Total 6 marks)

3.

(i) The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find the values of b and c, and show that $\mathbf{u}$ is a unit vector. **[6]**

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. **[3]**

(Total 9 marks)

4.

Prove that $n! > 2^n$ for $n \geq 4$. **[5]**

(Total 5 marks)

5.

Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. **[4]**

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[3]**

(iii) Calculate the volume of the pyramid $ABCD$. **[3]**

[The volume of the pyramid is $V = \frac{1}{3} \times$ base area $\times$ perpendicular height.]

(Total 10 marks)

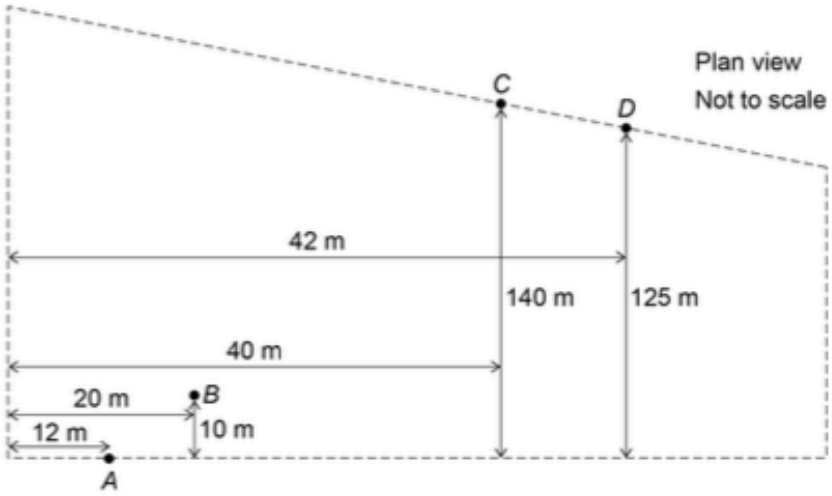
6.

A lighting engineer is setting up part of a display inside a large building. The diagram shows a plan view of the area in which he is working.

He has two lights, which project narrow beams of light.

One is set up at a point 3 metres above the point *A* and the beam from this light hits the wall 23 metres above the point *D*.

The other is set up 1 metre above the point *B* and the beam from this light hits the wall 29 metres above the point *C*.



(a) By creating a suitable model, show that the beams of light intersect.

[6 marks]

(b) Find the angle between the two beams of light.

[3 marks]

(c) State one way in which the model you created in part **(a)** could be refined.

[1 mark]

(Total 10 marks)