# Topic X2 Vectors and induction (Pre-TT A) [48] MARKSCHEME

1.

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|---|---|---|---|
| (i) | M1 | | Attempt at matrix multiplication |
| $\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$ | A1<br>A1 | 3 | Correct $\mathbf{A}^2$<br>Correct $\mathbf{A}^3$ |
| (ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$ | B1 | 1 | Sensible conjecture made |
| (iii) | B1<br>M1<br>A1<br>A1 | <br><br><br>4<br>8 | State that conjecture is true for $n$ = 1 or 2<br>Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa<br>Obtain correct matrix<br>Statement of induction conclusion |

2.

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| (i) | METHOD 1<br>Lines meet where<br>$(x =)\ k+2\lambda = k+\mu$<br>$(y =)\ -1-5\lambda = -4-4\mu$<br>$(z =)\ 1-3\lambda = -2\mu$ | M1<br>A1 | | For using parametric form to find where lines meet<br>For at least 2 correct equations |
| | | M1 | | For attempting to solve any 2 equations |
| | $\Rightarrow\ \lambda = -1,\ \ \mu = -2$ | A1 | | For correct values of $\lambda$ and $\mu$ |
| | | B1 | | For attempting a check in 3rd equation *OR* verifying point of intersection is on both lines |
| | $\Rightarrow\ (k-2, 4, 4)$ | A1 | 6 | For correct point of intersection (allow vector) |
| | | | | **SR** For finding $\lambda$ *OR* $\mu$ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1 |
| | METHOD 2<br>$d = \dfrac{\lvert [0, 3, 1] . [2, -5, -3] \times [1, -4, -2] \rvert}{\lvert \mathbf{b} \times \mathbf{c} \rvert}$ | | | For using $\mathbf{a} . \mathbf{b} \times \mathbf{c}$ with appropriate vectors (division by $\lvert \mathbf{b} \times \mathbf{c} \rvert$ is not essential) |
| | $d = c[0, 3, 1] . [-2, 1, -3] = 0$<br>$\Rightarrow$ lines intersect | B1 | | and showing $d = 0$ correctly |
| | Lines meet where<br>$(x =)\ (k+)\ 2\lambda = (k+)\ \mu$<br>$(y =)\ -1-5\lambda = -4-4\mu$<br>$(z =)\ 1-3\lambda = -2\mu$ | M1<br>A1 | | For using parametric form to find where lines meet<br>For at least 2 correct equations |
| | | M1 | | For attempting to solve any 2 equations |
| | $\Rightarrow\ \lambda = -1,\ \ \mu = -2$ | A1 | | For correct value of $\lambda$ *OR* $\mu$ |
| | $\Rightarrow\ (k-2, 4, 4)$ | A1 | | For correct point of intersection (allow vector) |
| | METHOD 3<br>e.g. $x-k = \dfrac{2(y+1)}{-5} = \dfrac{y+4}{-4}$ | M1 | | For solving one pair of simultaneous equations |
| | $\Rightarrow y = 4$ | A1 | | For correct value of $x$, $y$ or $z$ |
| | $\dfrac{z-1}{-3} = \dfrac{y+1}{-5}$ | M1 | | For solving for the third variable |
| | $x = k-2$ *OR* $z = 4$ | A1 | | For correct values of 2 of $x$, $y$ and $z$ |
| | $x-k = \dfrac{z}{-2}$ checks with $x = k-2,\ z = 4$ | B1 | | For attempting a check in 3rd equation |
| | $\Rightarrow\ (k-2, 4, 4)$ | A1 | | For correct point of intersection (allow vector) |

3.

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| **(i)** | Attempt scalar prod {**u**.(4**i** + **k**) or **u**.(4**i** + 3**j** + 2**k**)}= 0 | M1 | where **u** is the given vector |
| | Obtain $\frac{12}{13}+c=0$ or $\frac{12}{13}+3b+2c=0$ | A1 | |
| | $c=-\frac{12}{13}$ | A1 | |
| | $b=\frac{4}{13}$ | A1 | cao No ft |
| | Evaluate $\left(\frac{3}{13}\right)^2+(\text{their } b)^2+(\text{their } c)^2$ | M1 | Ignore non-mention of $\sqrt{\ }$ |
| | Obtain $\frac{9}{169}+\frac{144}{169}+\frac{16}{169}=1$ AG | A1 6 | Ignore non-mention of $\sqrt{\ }$ |

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| **(ii)** | Use $\cos\theta=\frac{x.y}{\lvert x\rvert\lvert y\rvert}$ | M1 | |
| | Correct method for finding scalar product | M1 | |
| | 36° (35.837653…) Accept 0.625 (rad) | A1 3 | From $\frac{18}{\sqrt{17}\sqrt{29}}$ |

**SR** If 4**i**+**k** = (4,1,0) in (i) & (ii), mark as scheme but allow final A1 for 31°(31.160968) or 0.544

**9**

4.

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| Let $n=4$, then<br>$4!=24$ and $2^4=16$ so $4!>2^4$ | B1 | 2.1 | Basis case for proof by induction | |
| Assume true for $n=r$<br>$r!>2^r$ for $r\geq 4$<br>Then for $n=r+1$ | M1 | 2.1 | Assumption | |
| $(r+1)!=(r+1)\times r!>(r+1)\times 2^r$ by assumption | M1 | 1.1 | Add next statement | |
| Since $r+1>2$, $(r+1)\times 2^r>2\times 2^r=2^{r+1}$<br>so $(r+1)!>2^{r+1}$ | E1 | 2.2a | Sufficient working to establish true for $r+1$ | Must state that $r+1>2$ oe |
| If true for $r$ then true for $r+1$.<br>Hence, given basis case, the statement is true for all positive integers. | E1 | 2.4 | Clear conclusion for induction process | A *formal* proof is required for full marks<br>Accept other *complete* methods |
| | [5] | | | |

5.

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| **(i)** | | <u>In each part, mark the answers, ignoring the labels</u><br>$AB=\sqrt{91}$; $AC=\sqrt{27}$ or $3\sqrt{3}$ ISW | B1; B1 | <u>To invoke MR, evidence must be clear</u><br>9.54 or 9.539392..; 5.2(0) or 5.1961524.. | |
| | | Attempting to use $\overrightarrow{AB}.\overrightarrow{AC}=AB.AC\cos\theta$ | M1 | or $BC^2=AB^2+AC^2-2AB.AC\cos\theta$ | |
| | | angle $BAC$ = 171 (3 sf) or 2.99 (rad) (3 sf) ISW | A1 | Final acute answer [8.68 or 0.152] /choice → A0 | 171 to 171.317 or 2.99 |
| | | | [4] | | |
| **(ii)** | | 6**i** + 4**j** – 2**k** or – 6**i** – 4**j** +2**k** | B1 | seen, irrespective of any labelling | |
| | | 6×(–1) + 4×(–3) – 2×(–9) = 0 (∴ perpendicular) **AG** | B1 | oe using (6,4,–2) or (– 6,–4,2) and… | ….(–1,–3,–9) or (1,3,9) |
| | | 6×1 + 4×1 – 2×5 = 0 (∴ perpendicular) **AG** | B1 | oe using (6,4,–2) or (– 6,–4,2) and… | … (1,1,5) or (–1,–1,–5) |
| | | | [3] | | |
| **(iii)** | | ($AD$ =) $\sqrt{56}$ or $2\sqrt{14}$ or 7.48… soi | B1 | | |
| | | area $ABC$= ½(their)$AB$ × (their)$AC$ × sin(their)$BAC$ | M1 | (✓= 3.74… but M mark, not A) | |
| | | $9.3\leq V<9.35$, $9\frac{1}{3}$ ISW | A1 | Accept even if (i) angle given as 8.68….. | i.e. the acute version not accepted in (i) |
| | | | [3] | | |

6.

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| (a) | Models light beams as straight lines and forms vector equations for straight lines using a suitable origin | AO3.3 | M1 | Modelling beams of light as straight lines taking the origin as point *A*: $\mathbf{r}_A = \begin{pmatrix}0\\0\\3\end{pmatrix} + \lambda\left(\begin{pmatrix}30\\125\\23\end{pmatrix} - \begin{pmatrix}0\\0\\3\end{pmatrix}\right)$ $= \begin{pmatrix}0\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}30\\125\\20\end{pmatrix}$ $\mathbf{r}_B = \begin{pmatrix}8\\10\\1\end{pmatrix} + \mu\left(\begin{pmatrix}28\\140\\29\end{pmatrix} - \begin{pmatrix}8\\10\\1\end{pmatrix}\right)$ $= \begin{pmatrix}8\\10\\1\end{pmatrix} + \mu\begin{pmatrix}20\\130\\28\end{pmatrix}$ $30\lambda = 8 + 20\mu$ $125\lambda = 10 + 130\mu$ $\lambda = \frac{3}{5}$ and $\mu = \frac{1}{2}$ $3 + \frac{3}{5} \times 20 = 15$ $1 + \frac{1}{2} \times 28 = 15$ $\therefore$ Intersect |
| | Forms correct vector equation for a line. Allow one slip | AO1.1b | A1 | |
| | Forms correct vector equation for second line. Allow one slip | AO1.1b | A1 | |
| | Forms equations for two components using 'their' model FT 'their' lines | AO3.4 | M1 | |
| | Solves 'their' equations correctly FT 'their' lines | AO1.1b | A1F | |
| | Checks with third component and concludes that the beams of light intersect<br>This mark is available only if all previous marks have been awarded | AO2.1 | R1 | |
| (b) | Evaluates scalar product for 'their' direction vectors. (PI) | AO3.1a | M1 | $\begin{pmatrix}30\\125\\20\end{pmatrix} \bullet \begin{pmatrix}20\\130\\28\end{pmatrix} = 17410$ $\cos\theta = \frac{17410}{\sqrt{30^2+125^2+20^2} \times \sqrt{20^2+130^2+28^2}}$ $\cos\theta = \frac{17410}{\sqrt{16925} \times \sqrt{18084}} = 0.9951$ $\theta = 5.6°$ |
| | Sets up equation to find angle. (PI) FT only if previous M1 awarded | AO1.1a | M1 | |
| | Obtains correct angle. | AO1.1b | A1 | |
| (c) | States appropriate refinement. | AO3.5c | E1 | Take account of the width of the beams. |
| | **Total** | | **10** | |