

Topic X2 Vectors and induction (Pre-TT B) [47]

1.

Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t \begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s \begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is a constant.

(i) Calculate the acute angle between the lines.

[5]

(ii) Given that these two lines intersect, find a and the point of intersection.

[8]

(Total 13 marks)

2.

(i) Find the value of k such that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix}$ are perpendicular. [2]

Two lines have equations $l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(ii) Find the point of intersection of l_1 and l_2 .

[4]

(iii) The vector $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$ is perpendicular to the lines l_1 and l_2 .

Find the values of a and b .

[5]

(Total 11 marks)

3.

Prove by induction that for all positive integers n ,

$$f(n) = 2^{3n-1} + 3(5^{2n-1})$$

is divisible by 17

(6)

(Total 6 marks)

4.

The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ . [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ . [4]

It is given that OT intersects PQ .

(iii) Find the position vector of the point of intersection of OT and PQ . [3]

(iv) Hence find the perpendicular distance from O to PQ , giving your answer in an exact form. [2]

(Total 11 marks)

5.

The equation of a straight line l is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

O is the origin.

(i) Find the position vector of the point P on l such that OP is perpendicular to l . [3]

(ii) A point Q on l is such that the length of OQ is 3 units. Find the two possible position vectors of Q . [3]

(Total 6 marks)