Topic X2 Vectors and induction (Pre-TT B) [47] MARKSCHEME

1.

(i)	Using $\begin{pmatrix}-8\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}-9\\2\\-5\end{pmatrix}$ as the relevant vectors	M1		i.e. correct direction vectors
	Using $\cos\theta=\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$ AEF for any 2 vectors	M1		Accept $\cos\theta=\left\lvert\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}\right\rvert$
	Method for scalar product of any 2 vectors	M1		
	Method for finding magnitude of any vector	M1		
	15° (15.38…), 0.268 rad	A1	5	
(ii)	Produce (at least) 2 of the 3 eqns in t and s	M1		e.g. $4-8t=-2-9s$, $-6-2t=-2-5s$
	Solve the (x) and (z) equations	M1		
	$t=3$ or $s=2$	A1		for first value found
	$s=2$ or $t=3$ f.t.	A1√		for second value found
	Substituting their (t,s) into (y) equation	M1		
	$a=1$	A1		
	Substituting their t into l_1 or their (s,a) into l_2	M1		
	$\begin{pmatrix}-20\\5\\-12\end{pmatrix}$	A1	8	Any format but not $\begin{pmatrix}\ \\ \ \\ \ \end{pmatrix}+\begin{pmatrix}\ \\ \ \\ \ \end{pmatrix}$

2.

(i)		$-1\times-2+2\times3+1\times k=0$	**M1**	1.1	Attempt the scalar product and set equal to zero soi	Allow use of **i, j, k** notation
		$\Rightarrow k=-4$	**A1** **[2]**	1.1		
(ii)		Equate x and y coordinates: $3+\lambda=6+2\mu\Rightarrow\lambda-2\mu=3$ $2-\lambda=5+\mu\Rightarrow\lambda+\mu=-3$	**M1**	2.1	Use coordinates to find μ and λ.	
		$\Rightarrow\mu=-2,\lambda=-1$	**A1**	1.1		
		Consistent with z coordinates since $7+3\times(-1)=4$ and $2-(-2)=4$	**E1**	1.1	Check consistency with third coordinate	
		So the point of intersection is $(2, 3, 4)$	**A1** **[4]**	1.1		
(iii)		The vector product find a mutual perpendicular $\begin{pmatrix}1\\-1\\3\end{pmatrix}\times\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}-2\\-7\\3\end{pmatrix}$	**M1** **A1**	3.1a 1.1	Attempt the vector product, by any valid method BC	OR **M1A1** $\begin{pmatrix}1\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\a\\b\end{pmatrix}=0$ and $\begin{pmatrix}2\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\a\\b\end{pmatrix}=0$
		$\begin{pmatrix}1\\a\\b\end{pmatrix}=\lambda\begin{pmatrix}-2\\-7\\3\end{pmatrix}$	**M1**	3.1a		**A1** $1-a+3b=0$ and $2+a-b=0$ **M1** Solve simultaneous equations
		$\lambda=-\frac{1}{2}$	**M1**	1.1		
		$a=3.5,\ b=-1.5$	**A1** **[5]**	1.1		**A1** $a=3.5,\ b=-1.5$

3.

When $n=1$, $2^{3n+1}+3\left(5^{2n+1}\right)=16+375=391$ $391 = 17 \times 23$ so the statement is true for $n = 1$	**B1**	2.2a
Assume true for $n = k$ so $2^{3k+1}+3\left(5^{2k+1}\right)$ is divisible by 17	**M1**	2.4
$\mathrm{f}(k+1)-\mathrm{f}(k)=2^{3k+4}+3\left(5^{2k+3}\right)-2^{3k+1}-3\left(5^{2k+1}\right)$	**M1**	2.1
$=7\times2^{3k+1}+7\times3\left(5^{2k+1}\right)+17\times3\left(5^{2k+1}\right)$		
$=7\mathrm{f}(k)+17\times3\left(5^{2k+1}\right)$	**A1**	1.1b
$\mathrm{f}(k+1)=8\mathrm{f}(k)+17\times3\left(5^{2k+1}\right)$	**A1**	1.1b
If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	**A1**	2.4
	(6)	

4.

(i) For (either point) + t(diff between posn vectors)	M1		"r =" not necessary for the M mark
r = (either point) + t(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**)	A1	2	… but it is essential for the A mark
(ii) **r** = s(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + s(**i** + 2**j** – **k**)	B1		Accept any parameter, including t
Eval scalar product of **i**+2**j**-**k** & their dir vect in (i)	M1		
Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3)	A1		This is just one example of numbers involved
= 0 <u>and</u> state perpendicular **AG**	A1	4	
(iii) For at least two equations with diff parameters	M1		e.g. $5+t=s,\ 2-2t=2s,\ -9-3t=-s$
Obtain $t=-2$ or $s=3$ (possibly -3 or 2 or -2)	A1		Check if $t = 2,1$ or -1
Subst. into eqn AB or OT and produce 3**i** + 6**j** – 3**k**	A1	3	
(iv) Indicate that $\left\|\overline{OC}\right\|$ is to be found	M1		where C is their point of intersection
$\sqrt{54}$;f.t. $\sqrt{a^2+b^2+c^2}$ from a**i** + b**j** + c**k** in (iii)	√A1	2	

<u>In the above question, accept any vectorial notation</u>
t and s may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.

5.

(i)		Use $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+t\\-t\\2\end{pmatrix}$	B1	
		$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\-1\\0\end{pmatrix}=0$	M1	
		$\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\2\end{pmatrix}$ or $\frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+2\mathbf{k}$	A1	
			[3]	
(ii)		$(1+t)^2+t^2+4=3^2$ or $\sqrt{(1+t)^2+t^2+4}=3$	M1	FT from their **(i)** P
		$t=1$ or -2	A1	SR If A0A0 award A1A0 for either value of t leading to its correct answer.
		$\begin{pmatrix}2\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\2\end{pmatrix}$	A1	
			[3]	