Topic X3 Mechanics AS (Post-TT A) [67]

1.

A car, of mass 1200 kg, moves on a straight horizontal road where it has a maximum speed of 40 m s^{-1}

When the car travels at a speed of v m s^{-1} it experiences a resistance force which can be modelled as being of magnitude $30v$ newtons.

(a) Show that the power output of the car is 48 000 W, when it is travelling at its maximum speed.

[3 marks]

(b) Find the maximum acceleration of the car when it is travelling at a speed of 25 m s^{-1}

[4 marks]

(Total 7 marks)

2.

A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

(Total 10 marks)

3.

A car of mass 800 kg experiences a resistance of magnitude kv^2 N, where k is a constant and v m s^{-1} is the car's speed. The car's engine is working at a constant rate of P W. At an instant when the car is travelling on a horizontal road with speed 20 m s^{-1} its acceleration is 0.75 m s^{-2}. At an instant when the car is ascending a hill of constant slope 12° to the horizontal with speed 10 m s^{-1} its acceleration is 0.25 m s^{-2}.

(i) Show that $k = 0.900$, correct to 3 decimal places, and find P. **[7]**

The power is increased to $1.5P$ W.

(ii) Calculate the maximum steady speed of the car on a horizontal road. **[3]**

(Total 10 marks)

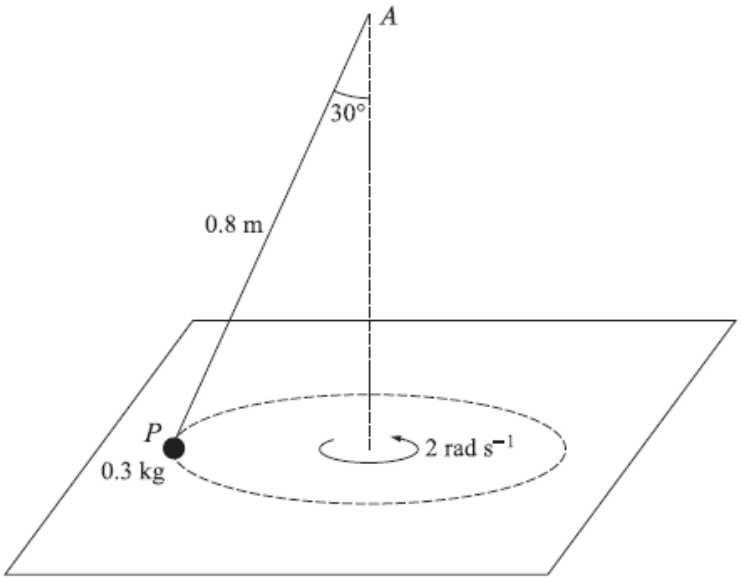
4.

The universal law of gravitation states that $F = \frac{Gm_1m_2}{r^2}$ where F is the magnitude of the force between two objects of masses m_1 and m_2 which are a distance r apart and G is a constant.

Find the dimensions of G. **[4]**

(Total 4 marks)

5.



One end of a light inextensible string of length 0.8 m is attached to a fixed point A which lies above a smooth horizontal table. The other end of the string is attached to a particle P, of mass 0.3 kg, which moves in a horizontal circle on the table with constant angular speed 2 rad s^{-1}. AP makes an angle of 30° with the vertical (see diagram).

(i) Calculate the tension in the string. [4]

(ii) Calculate the normal contact force between the particle and the table. [3]

The particle now moves with constant speed v m s^{-1} and is on the point of leaving the surface of the table.

(iii) Calculate v. [6]

(Total 13 marks)

6.

A particle P of mass $2m$ is moving on a smooth horizontal surface with speed u when it collides directly with a particle Q of mass km whose speed is $3u$ in the opposite direction. As a result of the collision, the directions of motion of both particles are reversed and the speed of P is halved.

(i) Find, in terms of u and k, the speed of Q after the collision. Hence write down the range of possible values of k. [4]

(ii) Calculate the magnitude of the impulse which Q exerts on P. [2]

(iii) Given that $k = \frac{1}{2}$, calculate the coefficient of restitution between P and Q. [3]

(Total 9 marks)

7.

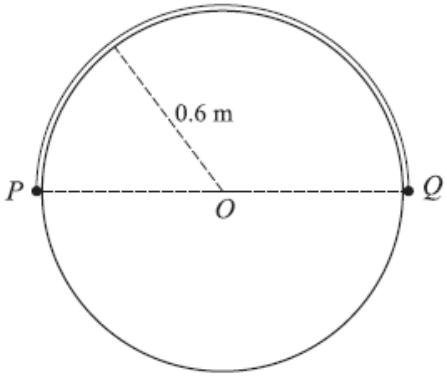


Fig. 1

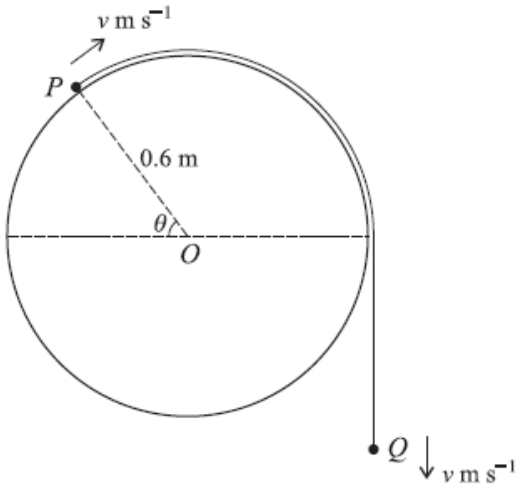


Fig. 2

A smooth horizontal cylinder of radius 0.6 m is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length 0.6π m has particles P and Q, of masses 0.3 kg and 0.4 kg respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, with the horizontal, the particles have speed v m s^{-1} (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04 \sin\theta.$$ [5]

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46 \sin\theta - 3.36\theta) \text{ newtons.}$$

Hence find the value of θ for which the magnitude of the contact force is greatest. [6]

(iii) Find the transverse component of the acceleration of P in terms of θ. [3]

(Total 14 marks)