Topic X3 Mechanics AS (Post-TT B) [45]

1.

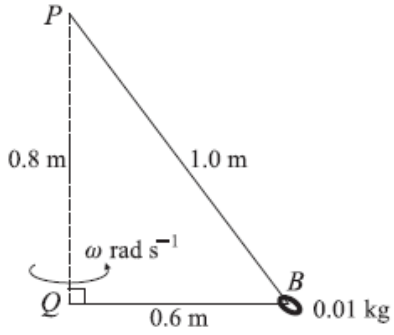
A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from 100 m s^{-1} to 150 m s^{-1} in a time t seconds.

(i) Calculate the value of t. [4]

(ii) Calculate the acceleration of the rocket at the instant when its speed is 120 m s^{-1}. [4]

(Total 8 marks)

2.



One end of a light inextensible string of length 1.6 m is attached to a point P. The other end is attached to the point Q, vertically below P, where $PQ = 0.8$ m. A small smooth bead B, of mass 0.01 kg, is threaded on the string and moves in a horizontal circle, with centre Q and radius 0.6 m. QB rotates with constant angular speed ω rad s^{-1} (see diagram).

(i) Show that the tension in the string is 0.1225 N. [3]

(ii) Find ω. [3]

(iii) Calculate the kinetic energy of the bead. [2]

(Total 8 marks)

3.

A box of mass 50 kg is dragged along a horizontal floor by a constant force of magnitude 400 N acting at an angle of α above the horizontal. The total resistance to the motion of the box has magnitude 300 N. The box starts from rest at the point O, and passes the point P, 25 m from O, with a speed of 2 m s^{-1}.

(i) For the box's motion from O to P, find

(a) the increase in kinetic energy of the box, [1]

(b) the work done against the resistance to motion of the box. [1]

(ii) Hence calculate α. [3]

(Total 5 marks)

4.

Marco is riding his bicycle at a constant speed of $12\,\text{m s}^{-1}$ along a horizontal road, working at a constant rate of 300 W. Marco and his bicycle have a combined mass of 75 kg.

(i) Calculate the wind resistance acting on Marco and his bicycle. [2]

Nicolas is riding his bicycle at the same speed as Marco and directly behind him. Nicolas experiences 30% less wind resistance than Marco.

(ii) Calculate the power output of Nicolas. [2]

The two cyclists arrive at the bottom of a hill which is at an angle of 1° to the horizontal. Marco increases his power output to 500 W.

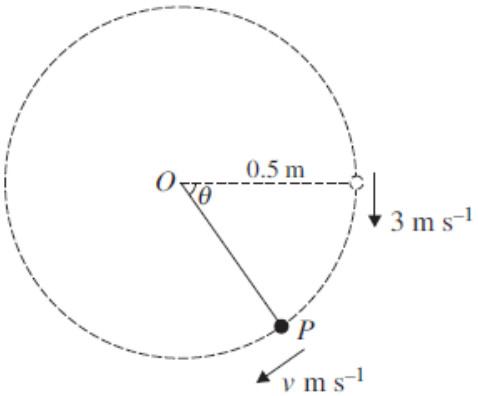
(iii) Assuming Marco's wind resistance is unchanged, calculate his instantaneous acceleration immediately after starting to climb the hill. [5]

Marco reaches the top of the hill at a speed of $13\,\text{m s}^{-1}$. He then freewheels down a hill of length 200 m which is at a constant angle of 10° to the horizontal. He experiences a constant wind resistance of 120 N.

(iv) Calculate Marco's speed at the bottom of this hill. [5]

(Total 14 marks)

5.



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. With the string taut and horizontal, P is projected with a velocity of $3\,\text{m s}^{-1}$ vertically downward. P begins to move in a vertical circle with centre O. While the string remains taut the angular displacement of OP is θ radians from its initial position, and the speed of P is $v\,\text{m s}^{-1}$ (see diagram).

(i) Show that $v^2 = 9 + 9.8\sin\theta$. **[3]**

(ii) Find, in terms of θ, the radial and tangential components of the acceleration of P. **[3]**

(iii) Show that the tension in the string is $(3.6 + 5.88\sin\theta)$ N and hence find the value of θ at the instant when the string becomes slack, giving your answer correct to 1 decimal place. **[4]**

(Total 10 marks)