Topic X3 Mechanics AS (Pre-TT A) [60]

1.

A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of 35° with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

(Total 3 marks)

2.

A small sphere of mass 0.3 kg is dropped from rest at a height of 2 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.4 m above the ground. Ignoring air resistance, calculate the magnitude of the impulse which the ground exerts on the sphere when it rebounds. [5]

(Total 5 marks)

3.

A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

(Total 6 marks)

4.

A tank full of liquid has a hole made in its base.

Two students, Sarah and David, propose two different models for the speed, v, at which liquid exits the tank.

David thinks that v will depend on the height of the liquid in the tank, h, the acceleration due to gravity, g, and the density of the liquid, ρ, such that $v \propto g^a h^b \rho^c$ where a, b and c are constants.

Sarah thinks that v will not depend on the density of the liquid and suggests the model $v \propto g^a h^b$

(a) By considering dimensions, explain which student's model should be rejected.

[2 marks]

(b) Find the values of the constants in order for the model that you did **not** reject in part **(a)** to be dimensionally consistent.

[2 marks]

(Total 4 marks)

5.

A model train has mass 100 kg. When the train is moving with speed $v\,\mathrm{m\,s^{-1}}$ the resistance to its motion is $3v^2$ N and the power output of the train is $\frac{3000}{v}$ W.

(i) Show that the driving force acting on the train is 120 N at an instant when the train is moving with speed $5\,\mathrm{m\,s^{-1}}$. [2]

(ii) Find the acceleration of the train at an instant when it is moving horizontally with speed $5\,\mathrm{m\,s^{-1}}$. [2]

The train moves with constant speed up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{98}$.

(iii) Calculate the speed of the train. [5]

6.

Three smooth spheres A, B and C, of equal radius and of masses $3m$ kg, $2m$ kg and m kg respectively, are free to move in a straight line on a smooth horizontal table. Spheres B and C are stationary. Sphere A is moving with speed $2\,\mathrm{m\,s^{-1}}$ when it collides directly with sphere B. The collision is perfectly elastic.

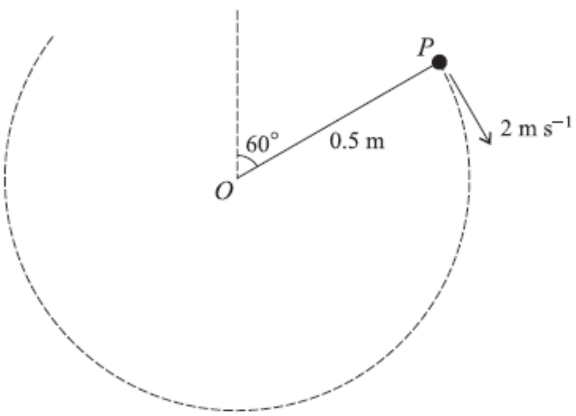
(i) Find the velocities of A and B after the collision. [6]

(ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B continues its motion and hits C. After the collision, B continues in the same direction with speed $1.0\,\mathrm{m\,s^{-1}}$ and C moves with speed $2.8\,\mathrm{m\,s^{-1}}$.

(iii) Find the coefficient of restitution between B and C. [2]

7.



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. With the string taut and at an angle of 60° to the upward vertical, P is projected with speed $2\,\mathrm{m\,s^{-1}}$ (see diagram). P begins to move without air resistance in a vertical circle with centre O. When the string makes an angle θ with the upward vertical, the speed of P is $v\,\mathrm{m\,s^{-1}}$.

(i) Show that $v^2 = 8.9 - 9.8\cos\theta$. [4]

(ii) Find the tension in the string in terms of θ. [4]

(iii) P does not move in a complete circle. Calculate the angle through which OP turns before P leaves the circular path. [4]

8.

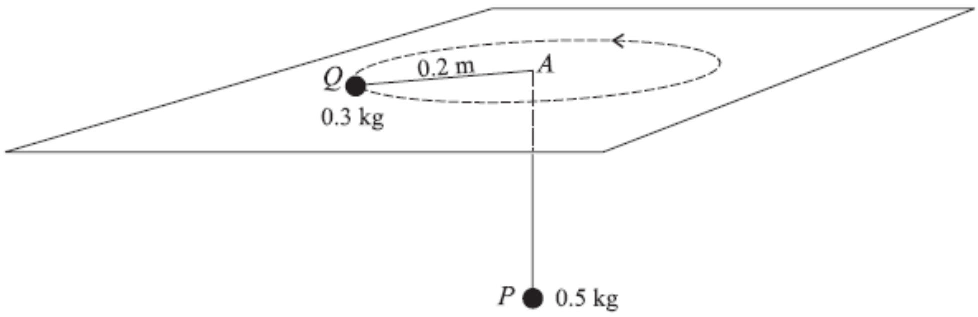


Fig. 1

A light inextensible string of length 1 m passes through a small smooth hole A in a fixed smooth horizontal plane. One end of the string is attached to a particle P, of mass 0.5 kg, which hangs in equilibrium below the plane. The other end of the string is attached to a particle Q, of mass 0.3 kg, which rotates with constant angular speed in a circle of radius 0.2 m on the surface of the plane (see Fig. 1).

(i) Calculate the tension in the string and hence find the angular speed of Q. [4]

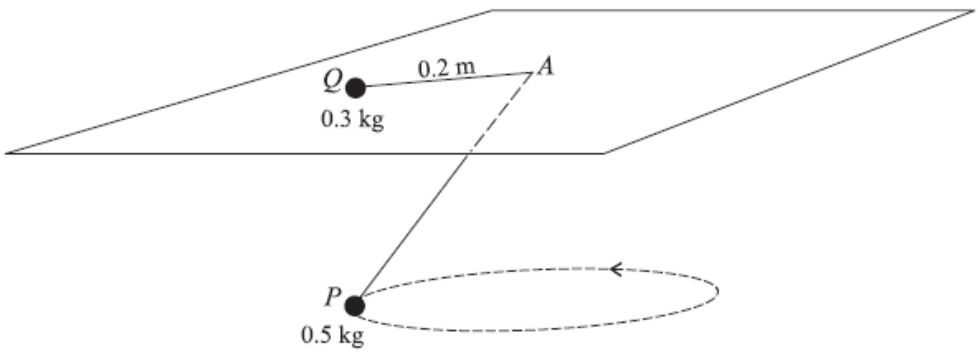


Fig. 2

The particle Q on the plane is now fixed to a point 0.2 m from the hole at A and the particle P rotates in a horizontal circle of radius 0.2 m (see Fig. 2).

(ii) Calculate the tension in the string. [4]

(iii) Calculate the speed of P. [3]

(Total 11 marks)