Topic X3 Mechanics AS (Pre-TT B) [53]

1.

A car of mass 1250 kg experiences a resistance to its motion of magnitude kv^2 N, where k is a constant and v m s^{-1} is the car's speed. The car travels in a straight line along a horizontal road with its engine working at a constant rate of P W. At a point A on the road the car's speed is 15 m s^{-1} and it has an acceleration of magnitude 0.54 m s^{-2}. At a point B on the road the car's speed is 20 m s^{-1} and it has an acceleration of magnitude 0.3 m s^{-2}.

(i) Find the values of k and P. **[7]**

The power is increased to 15 kW.

(ii) Calculate the maximum steady speed of the car on a straight horizontal road. **[3]**

(Total 10 marks)

2.

A car of mass 700 kg is travelling up a hill which is inclined at a constant angle of 5° to the horizontal. At a certain point P on the hill the car's speed is 20 m s^{-1}. The point Q is 400 m further up the hill from P, and at Q the car's speed is 15 m s^{-1}.

(i) Calculate the work done by the car's engine as the car moves from P to Q, assuming that any resistances to the car's motion may be neglected. [4]

Assume instead that the resistance to the car's motion between P and Q is a constant force of magnitude 200 N.

(ii) Given that the acceleration of the car at Q is zero, show that the power of the engine as the car passes through Q is 12.0 kW, correct to 3 significant figures. [3]

(iii) Given that the power of the car's engine at P is the same as at Q, calculate the car's retardation at P. [3]

(Total 10 marks)

3.

Two small spheres A and B are moving towards each other along a straight line on a smooth horizontal surface. A has speed 3 m s^{-1} and B has speed 1.5 m s^{-1} before they collide directly. The direction of motion of B is reversed in the collision. The speeds of A and B after the collision are 2 m s^{-1} and 2.9 m s^{-1} respectively.

(i) (a) Show that the direction of motion of A is unchanged by the collision. **[2]**

(b) Calculate the coefficient of restitution between A and B. **[2]**

The mass of B is 0.2 kg.

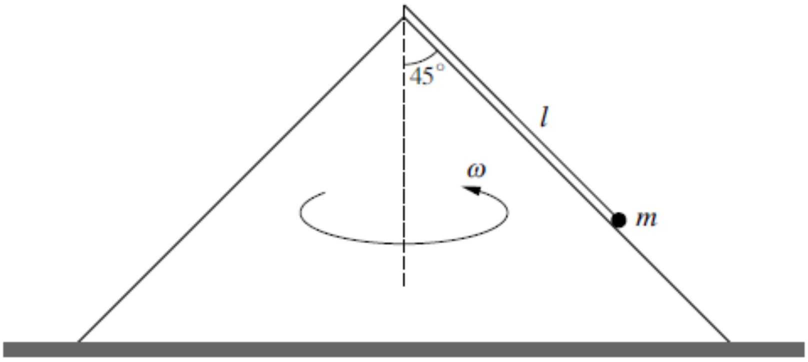
(ii) Find the mass of A. **[3]**

B continues to move at 2.9 m s^{-1} and strikes a vertical wall at right angles. The wall exerts an impulse of magnitude 0.68 N s on B.

(iii) Calculate the coefficient of restitution between B and the wall. **[4]**

(Total 11 marks)

4.



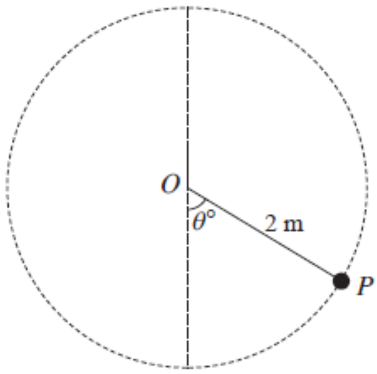
One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45°. The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R.

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. **[6]**

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. **[4]**

(Total 10 marks)

5.



A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point O. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of P is $7\ \text{m s}^{-1}$.

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ. [8]

(Total 12 marks)