Topic Y1 Complex numbers and roots of equations (Post-TT A) [48]

1.

Express $\dfrac{2+3\text{i}}{5-\text{i}}$ in the form $x + \text{i}y$, showing clearly how you obtain your answer. **[4]**

(Total 4 marks)

2.

Given that z is a complex number and that z^* is the complex conjugate of z

Prove that $zz^* - |z|^2 = 0$

[3 marks]

(Total 3 marks)

3.

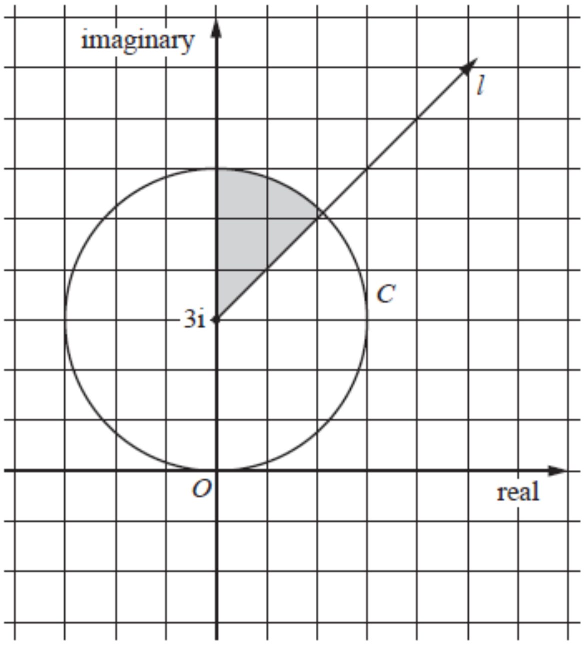
The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots $(3 + \text{i})$ and 2.

(i) Write down the other root of the equation. [1]

(ii) Find the values of a, b and c. [6]

(Total 7 marks)

4.



The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

(Total 7 marks)

5.

Use an algebraic method to find the square roots of $11 + (12\sqrt{5})i$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(Total 6 marks)

6.

The loci C_1 and C_2 are given by $|z - 3| = 3$ and $\arg(z - 1) = \frac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \text{ and } 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi.$$ [2]

(Total 8 marks)

7.

(i) Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

(Total 8 marks)

8.

$$f(x) = kx^2 + 3x - 11 \qquad g(x) = mx^3 - 2x^2 + 3x - 9$$

where k and m are real constants.

Given that

- the sum of the roots of f is equal to the product of the roots of g
- g has at least one root on the imaginary axis

(a) solve completely

(i) $f(x) = 0$

(ii) $g(x) = 0$

(7)

(b) Plot the roots of f and the roots of g on a single Argand diagram.

(2)

(Total 9 marks)