Topic Y1 Complex numbers and roots of equations (Post-TT A) [48] MS

1.

$\frac{7}{26} + \frac{17}{26}$ i.	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator

2.

Defines generalised z and z^* in Cartesian or polar form	AO1.2	B1	Let $z = a + bi$ then $z^* = a - bi$
Expands and simplifies zz^* and $\|z\|^2$ (at least one correct)	AO1.1b	M1	$zz^* - \|z\|^2 = (a+bi)(a-bi) - \left(\sqrt{a^2+b^2}\right)^2$ $= a^2 + abi - abi - (bi)^2 - \left(a^2+b^2\right)$ $= a^2 + b^2 - \left(a^2+b^2\right)$ $= 0$ **AG**
Completes a well structured argument to prove the required result. **AG** Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	ALT Let $z = re^{i\theta}$ then $z^* = re^{-i\theta}$ $zz^* - \|z\|^2 = re^{i\theta} re^{-i\theta} - r^2$ $= r^2 e^{i\theta - i\theta} - r^2$ $= r^2 - r^2$ $= 0$ **AG**

3.

(i)	$3 - i$	**B1**	Conjugate stated
		1	
(ii)	*EITHER*	**M1**	Use sum of roots
		A1	Obtain correct answer
		M1	Use sum of pairs of roots
		A1	Obtain correct answer
		M1	Use product of roots
	$a = -8,\ b = 22,\ c = -20$	**A1**	Obtain correct answers
		6	
	OR	**M1**	Attempt to find a quadratic factor
		A1	Obtain correct factor
		M1	Expand linear and quadratic factors
	$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
	OR		
		M1	Substitute 1 imaginary & the real root into eqn
		M1	Equate real and imaginary parts
		M1	Attempt to solve 3 eqns.
	$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers

4.

(i)		$\arg(z-3i)=\frac{1}{4}\pi$	M1 A1	Use arg $(z-a)=\theta$ in equation for l condone missing brackets Obtain correct answer
		$\lvert z-3i\rvert=3$	M1 A1 [4]	Use $\lvert z-a\rvert=k$ in equation for C, k must be real Obtain correct answer
(ii)		$\lvert z-3i\rvert\le 3$ or e.g. $x^2+(y-3)^2\le 9$ $\frac{1}{4}\pi\le\arg(z-3i)\le\frac{1}{2}\pi$ or $y\ge x+3,\ x\ge 0$	B1 B1 B1 [3]	Obtain correct inequality, or answer consistent with sensible **(i)** Each correct single inequality, or answer consistent with sensible **(i)** **SC if < used consistently, but otherwise all correct, B2**

5.

		$x^2-y^2=11$ and $xy=6\sqrt{5}$	M1 A1	Attempt to equate real and imaginary parts of $(x+iy)^2$ and $11+12\sqrt{5}$ Obtain both results cao
		$\pm(2\sqrt{5}+3i)$	M1* DM1 A1 A1 [6]	Obtain a quadratic in x^2 or y^2 Solve a 3 term quadratic to obtain a value for x or y Obtain 1 correct answer as complex number Obtain only the other correct answer

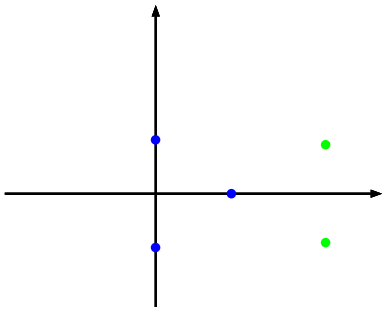
6.

(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only	B1B1 B1 B1 B1 B1		Sketch showing correct features N.B. treat 2 diagrams asa MR
(ii) Inside circle, below line, above x-axis	B2ft	6 2 **8**	Sketch showing correct region SR: B1ft for any 2 correct features

7.

(i) $\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	M1 A1	2	Correct binomial expansion seen Obtain given answer with no errors seen
(ii) *Either* $\alpha+\beta=5, \alpha\beta=7$	B1 B1		State or use correct values
$\alpha^3+\beta^3=20$	M1 A1		Find numeric value for $\alpha^3+\beta^3$ Obtain correct answer
$x^2-20x+343=0$	M1 A1ft	6 **8**	Use new sum and product correctly in quadratic expression Obtain correct equation
Or $u^{\frac{2}{3}}-5u^{\frac{1}{3}}+7=0$	M1 A1 M2 A2		Substitute $x=u^{\frac{1}{3}}$ Obtain correct answer Complete method for removing fractional powers Obtain correct answer
$u^3-20u+343=0$			

8.

Question	Scheme	Marks	AOs
6(a)	Attempts sum of roots of f (= −3/*k*) and product of roots of g (= 9/*m*) and uses them to form a relationship between *k* and *m.*	M1	3.1a
	So −3/*k* = 9/*m*	A1	1.1b
	Sum of roots of g is 2/*m* ⇒ 2/*m* is a root of g as the other roots have no real part. OR root on imaginary axis has form αi, and substituting in g and equating real and imaginary terms gives $2\alpha^2 - 9 = 0$ & $3\alpha - m\alpha^3 = 0$	B1	3.1a
	g(2/*m*) = 0 ⇒ *m*(2/*m*)^3 − 2(2/*m*)^2 + 3(2/*m*) − 9 = 0 ⇒ *m*=… (*m* = 2/3) OR $\alpha^2 = \frac{9}{2} \neq 0 \Rightarrow m = \frac{3}{\alpha^2} = \ldots \left(= \frac{2}{3}\right)$	M1	1.1a
	So g(*x*) = 0 ⇒ $\left(\frac{2}{3}(x-3)\left(x^2 + \frac{9}{2}\right) = 0 \Rightarrow\right)$ $x = 3, \pm\frac{3\sqrt{2}}{2}i$	M1	1.1b
	k = −2/9 , f(*x*) = 0 ⇒ $x = \frac{-3 \pm \sqrt{3^2 - 4(-2/9)(-11)}}{2(-2/9)} = \ldots$	M1	2.2a
	$x = 3, \pm\frac{3\sqrt{2}}{2}i, \frac{27}{4} \pm \frac{3\sqrt{7}}{4}i$	A1	1.1b
		(7)	
(b)	 Correct roots for f plotted (shown green). Correct roots for g plotted (shown blue).	B1ft B1ft	1.1b 1.1b
		(2)	
			(9 marks)