

Topic Y1 Complex numbers and roots of equations (Post-TT A) [48] MS

1.

$\frac{7}{26} + \frac{17}{26}i$	M1 A1 A1 A1	4 4	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
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2.

Defines generalised z and z^* in Cartesian or polar form	AO1.2	B1	Let $z = a + bi$ then $z^* = a - bi$ $zz^* - z ^2 = (a + bi)(a - bi) - (\sqrt{a^2 + b^2})^2$ $= a^2 + abi - abi - (bi)^2 - (a^2 + b^2)$ $= a^2 + b^2 - (a^2 + b^2)$ $= 0$ AG <div style="background-color: #e0e0e0; padding: 5px;"> ALT Let $z = re^{i\theta}$ then $z^* = re^{-i\theta}$ $zz^* - z ^2 = re^{i\theta}re^{-i\theta} - r^2$ $= r^2e^{i\theta-i\theta} - r^2$ $= r^2 - r^2$ $= 0$ AG </div>
Expands and simplifies zz^* and $ z ^2$ (at least one correct)	AO1.1b	M1	
Completes a well structured argument to prove the required result. AG Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	

3.

(i) $3 - i$	B1 1	Conjugate stated
(ii) <i>EITHER</i>		
$a = -8, b = 22, c = -20$		M1 Use sum of roots A1 Obtain correct answer M1 Use sum of pairs of roots A1 Obtain correct answer M1 Use product of roots A1 Obtain correct answers
<i>OR</i>	6	
$a = -8, b = 22, c = -20$		M1 Attempt to find a quadratic factor A1 Obtain correct factor M1 Expand linear and quadratic factors A1A1A1 Obtain correct answers
<i>OR</i>		
$a = -8, b = 22, c = -20$		M1 Substitute 1 imaginary & the real root into eqn M1 Equate real and imaginary parts M1 Attempt to solve 3 eqns. A1A1A1 Obtain correct answers

4.

(i)	$\arg(z - 3i) = \frac{1}{4}\pi$ $ z - 3i = 3$	M1 A1 M1 A1 [4]	Use $\arg(z - a) = \theta$ in equation for l condone missing brackets Obtain correct answer Use $ z - a = k$ in equation for C , k must be real Obtain correct answer
(ii)	$ z - 3i \leq 3$ or e.g. $x^2 + (y - 3)^2 \leq 9$ $\frac{1}{4}\pi \leq \arg(z - 3i) \leq \frac{1}{2}\pi$ or $y \geq x + 3, x \geq 0$	B1 B1 B1 [3]	Obtain correct inequality, or answer consistent with sensible (i) Each correct single inequality, or answer consistent with sensible (i) SC if < used consistently, but otherwise all correct, B2

5.

	EITHER $a = -8$ $b = 25$ OR $a = -8$ $b = 25$	M1 A1 M1 A1 [4] M1 M1 A1 A1	Use sum of root and conjugate Obtain correct answer Use product of root and conjugate Obtain correct answer Substitute $4 + 3i$ or conjugate into equation Equate real and imaginary parts Obtain correct answer Obtain correct answer
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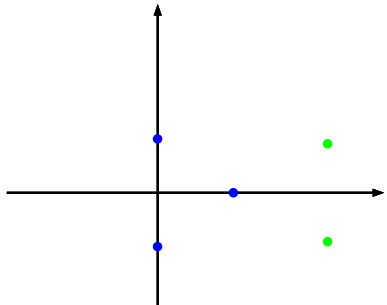
6.

(i) Circle, centre (3, 0), y -axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1 st quadrant only	B1B1 B1 B1 B1 B1		Sketch showing correct features N.B. treat 2 diagrams as MR
(ii) Inside circle, below line, above x -axis	B2ft	6 2 8	Sketch showing correct region SR: B1ft for any 2 correct features

7.

(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	M1 A1	2	Correct binomial expansion seen Obtain given answer with no errors seen
(ii) Either $\alpha + \beta = 5, \alpha\beta = 7$ $\alpha^3 + \beta^3 = 20$	B1 B1 M1 A1		State or use correct values Find numeric value for $\alpha^3 + \beta^3$ Obtain correct answer
$x^2 - 20x + 343 = 0$	M1 A1ft	6 8	Use new sum and product correctly in quadratic expression Obtain correct equation
Or $u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$ $u^3 - 20u + 343 = 0$	M1 A1 M2 A2		Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer Complete method for removing fractional powers Obtain correct answer

8.

Question	Scheme	Marks	AOs	
<p>6(a)</p>	<p>Attempts sum of roots of $f (= -3/k)$ and product of roots of $g (= 9/m)$ and uses them to form a relationship between k and m.</p>	M1	3.1a	
	<p>So $-3/k = 9/m$</p>	A1	1.1b	
	<p>Sum of roots of g is $2/m \Rightarrow 2/m$ is a root of g as the other roots have no real part. OR root on imaginary axis has form αi, and substituting in g and equating real and imaginary terms gives $2\alpha^2 - 9 = 0$ & $3\alpha - m\alpha^3 = 0$</p>	B1	3.1a	
	<p>$g(2/m) = 0 \Rightarrow m(2/m)^3 - 2(2/m)^2 + 3(2/m) - 9 = 0 \Rightarrow m = \dots$ ($m = 2/3$) OR $\alpha^2 = \frac{9}{2} \neq 0 \Rightarrow m = \frac{3}{\alpha^2} = \dots \left(= \frac{2}{3} \right)$</p>	M1	1.1a	
	<p>So $g(x) = 0 \Rightarrow \left(\frac{2}{3}(x-3) \left(x^2 + \frac{9}{2} \right) = 0 \Rightarrow \right) x = 3, \pm \frac{3\sqrt{2}}{2}i$</p>	M1	1.1b	
	<p>$k = -2/9, f(x) = 0 \Rightarrow x = \frac{-3 \pm \sqrt{3^2 - 4(-2/9)(-11)}}{2(-2/9)} = \dots$</p>	M1	2.2a	
	<p>$x = 3, \pm \frac{3\sqrt{2}}{2}i, \frac{27}{4} \pm \frac{3\sqrt{7}}{4}i$</p>	A1	1.1b	
		(7)		
<p>(b)</p>		<p>Correct roots for f plotted (shown green). Correct roots for g plotted (shown blue).</p>	<p>B1ft B1ft</p>	<p>1.1b 1.1b</p>
	(2)			
(9 marks)				