Topic Y1 Complex numbers and roots of equations (Post-TT B) [53]

1.

The complex numbers $3 - 2i$ and $2 + i$ are denoted by z and w respectively. Find, giving your answers in the form $x + iy$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(iz)^2$, [3]

(iii) $\frac{z}{w}$. [3]

(Total 8 marks)

2.

The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. **[4]**

(Total 4 marks)

3.

Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - i| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

(Total 6 marks)

4.

The complex number z satisfies the equation $z + 2iz^* = 12 + 9i$. Find z, giving your answer in the form $x + iy$. **[5]**

(Total 5 marks)

5.

In the cubic equation $4z^3 + az^2 + bz + c = 0$, a, b and c are real numbers. One root is $1 + \frac{3}{2}i$ and the sum of the roots is 6. Find the values of a, b and c. **[7]**

(Total 7 marks)

6.

The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

(Total 9 marks)

7.

(i) Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

(Total 14 marks)