Topic Y1 Complex numbers and roots of equations (Pre-TT A) [52]

1.

(i) Express $(1 + 8\mathrm{i})(2 - \mathrm{i})$ in the form $x + \mathrm{i}y$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\dfrac{1 + 8\mathrm{i}}{2 + \mathrm{i}}$ in the form $x + \mathrm{i}y$. [3]

(Total 5 marks)

2.

$$\mathrm{f}(z) = z^3 + pz^2 + qz - 15$$

where p and q are real constants.

Given that the equation $\mathrm{f}(z) = 0$ has roots

$$\alpha, \ \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $\mathrm{f}(z) = 0$

(5)

(b) Hence find the value of p.

(2)

(Total 7 marks)

3.

Use an algebraic method to find the square roots of the complex number $15 + 8\mathrm{i}$. [6]

(Total 6 marks)

4.

The loci C_1 and C_2 are given by

$$|z - 2\mathrm{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \mathrm{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

(Total 7 marks)

5.

The complex number $2 - \mathrm{i}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. [2]

(ii) Given that $az + bz^* = 4 - 8\mathrm{i}$, find the values of the real constants a and b. [5]

(Total 7 marks)

6.

The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

(Total 5 marks)

7.

$$\mathrm{p}(z) = z^4 + 3z^2 + az + b,\ a \in \mathbb{R}, b \in \mathbb{R}$$

$2 - 3\mathrm{i}$ is a root of the equation $\mathrm{p}(z) = 0$

(a) Express $\mathrm{p}(z)$ as a product of quadratic factors with real coefficients.

[5 marks]

(b) Solve the equation $\mathrm{p}(z) = 0$.

[1 mark]

(Total 6 marks)

8.

(a) Show on an Argand diagram the locus of points given by

$$|z - 10 - 12\mathrm{i}| = 8$$

(2)

Set A is defined by

$$A = \left\{z : 0 \leqslant \arg(z - 10 - 10\mathrm{i}) \leqslant \frac{\pi}{2}\right\} \cap \left\{z : |z - 10 - 12\mathrm{i}| \leqslant 8\right\}$$

(b) Shade the region defined by A on your Argand diagram.

(2)

(c) Determine the area of the region defined by A.

(5)

(Total 9 marks)