Topic Y1 Complex numbers and roots of equations (Pre-TT A) [52] MS

1.

(i) $2 + 16i - i - 8i^2$ $10 + 15i$	M1 A1	2	Attempt to multiply correctly Obtain correct answer
(ii)	M1 A1		Multiply numerator & denominator by conjugate Obtain denominator 5
$\frac{1}{5}(10 + 15i)$ or $2 + 3i$	A1ft	3	Their part (i) or 10 + 15i derived again / 5
		5	

2.

Question	Scheme	Marks
1(a)	$\alpha\left(\frac{5}{\alpha}\right)\left(\alpha + \frac{5}{\alpha} - 1\right) = 15$	M1 A1
	$\Rightarrow 5\alpha + \frac{25}{\alpha} - 5 = 15 \Rightarrow \alpha^2 - 4\alpha + 5 = 0$ $\Rightarrow \alpha = \frac{--4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$ or $(\alpha - 2)^2 - 4 + 5 = 0 \Rightarrow \alpha = ...$	M1
	$\Rightarrow \alpha = 2 \pm i$	A1
	Hence the roots of $f(z) = 0$ are $2 + i, 2 - i$ and 3	A1
		(5)
(b)	$p = -(\text{"}(2+i)\text{"} + \text{"}(2-i)\text{"} + \text{"}3\text{"}) \Rightarrow p = ...$	M1
	$\Rightarrow p = -7$ cso	A1
		(2)
	1(b) alternative	
	$f(z) = (z - 3)(z^2 - 4z + 5) \Rightarrow p = ...$	M1
	$\Rightarrow p = -7$ cso	A1
		(2)

3.

	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and $15 + 8i$
$x^2 - y^2 = 15$ and $xy = 4$	A1 A1		Obtain each result
	M1		Eliminate to obtain a quadratic in x^2 or y^2
	DM1		Solve to obtain $x = (\pm)4$, or $y = (\pm)1$
$\pm(4 + i)$	A1	6	Obtain only correct two answers as complex numbers
		6	

4.

(i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope	B1 B1 B1 B1 B1	 5	Sketch(s) showing correct features, each mark independent
(ii) 0 or 0 +0i and 2 + 2i	B1ftB1ft	2 **7**	Obtain intersections as complex numbers

5.

(i)		$\|z\|=\sqrt{5}$ $\arg z=-26.6^{\circ}$ or -0.464	B1 B1 [2]	Allow 2.2 Allow -27° or $-0.46(3)$
(ii)		 $a+b=2, b-a=-8$ $a=5,\ b=-3$	B1 M1 A1 M1 A1 [5]	$z^*=2+i$ stated or used Obtain two equations from real and imaginary parts Obtain correct equations Attempt to solve 2 linear equations Obtain correct answers

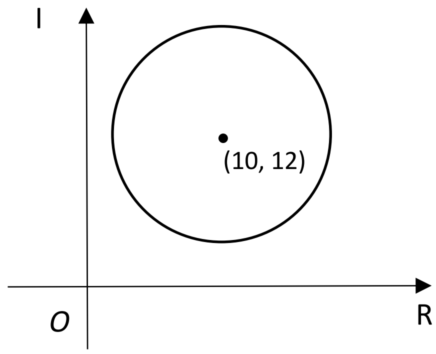
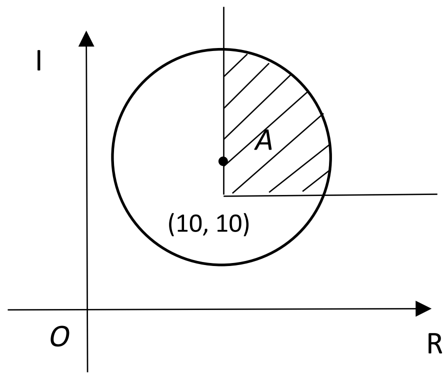
6.

$\{w=x-1\Rightarrow\}\ x=w+1$	**B1**
$(w+1)^3+3(w+1)^2-8(w+1)+6=0$	**M1**
$w^3+3w^2+3w+1+3(w^2+2w+1)-8w-8+6=0$	
$w^3+6w^2+w+2=0$	**M1** **A1** **A1**
	(5)
Alternative	
$\alpha+\beta+\gamma=-3, \alpha\beta+\beta\gamma+\alpha\gamma=-8, \alpha\beta\gamma=-6$	**B1**
sum roots $=\alpha-1+\beta-1+\gamma-1$	**M1**
$=\alpha+\beta+\gamma-3=-3-3=-6$	
pair sum $=(\alpha-1)(\beta-1)+(\alpha-1)(\gamma-1)+(\beta-1)(\gamma-1)$	
$=\alpha\beta+\alpha\gamma+\beta\gamma-2(\alpha+\beta+\gamma)+3$	
$=-8-2(-3)+3=1$	
product $=(\alpha-1)(\beta-1)(\gamma-1)$	
$=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$	
$=-6-(-8)-3-1=-2$	
$w^3+6w^2+w+2=0$	**M1** **A1** **A1**

7.

(a)	Makes a correct deduction about another root (PI)	AO2.2a	B1	$(z-(2-3i))(z-(2+3i)) = z^2-4z+13$
	Finds quadratic factor by expanding brackets or using sum and product of roots	AO1.1a	M1	$\therefore$ $\mathrm{p}(z) = (z^2-4z+13)(z^2+cz+d)$ $(z^2-4z+13)(z^2+cz+d) \equiv z^4+3z^2+az+b$
	Finds a correct quadratic factor	AO1.1b	A1	
	Compares coefficients with quartic z^4+3z^2+az+b	AO1.1a	M1	$c-4=0$ $13-4c+d=3$ $\Rightarrow c=4,\ d=6$
	States the correct product of quadratic factors	AO1.1b	A1	$\therefore$ $\mathrm{p}(z) = \left(z^2-4z+13\right)\left(z^2+4z+6\right)$
(b)	States all four correct solutions FT 'their' two quadratic factors from part **(a)** provided both M1 marks have been awarded	AO1.1b	B1F	$z = 2\pm 3\mathrm{i}, -2\pm\sqrt{2}\mathrm{i}$

8.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="3">3(a)</td><td rowspan="2">
</td><td>M1</td><td>1.1b</td></tr>
<tr><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td rowspan="2">
</td><td>B1</td><td>1.1b</td></tr>
<tr><td>B1ft</td><td>1.1b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="6">(c)</td><td>$8^2 - 2^2 = h^2 \Rightarrow h = ...$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>$h = 2\sqrt{15}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td>Triangle area $= \frac{1}{2} \times 2 \times 2\sqrt{15}$</td><td>M1</td><td>2.1</td></tr>
<tr><td>Sector area $= \frac{1}{2} \times 8^2 \times \left(\pi - \tan^{-1}\left(\sqrt{15}\right)\right)$ or $\frac{1}{2} \times 8^2 \times \left(\pi - \cos^{-1}\left(\frac{1}{4}\right)\right)$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Total area $= \frac{1}{2} \times 8^2 \times \left(\pi - \tan^{-1}\left(\sqrt{15}\right)\right) + \frac{1}{2} \times 2 \times 2\sqrt{15}$
$= 66.1$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(5)</td><td></td></tr>
<tr><td colspan="4">(9 marks)</td></tr>
<tr><td colspan="4">Notes</td></tr>
<tr><td colspan="4">(a) M1: A circle drawn
A1: A circle entirely in the first quadrant with the centre marked at (10, 12)
(b) B1: Correct pair of rays added to their diagram
B1ft: Area between their rays and within the circle shaded
(c) M1: Correct strategy to find the base (or angle) of the triangular part
A1: Correct length or angle
M1: Correct method for the area of the triangle
M1: Correct strategy for the area of the sector
A1: Correct answer (awrt 66.1)</td></tr>
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