Topic Y1 Complex numbers and roots of equations (Pre-TT B) [46]

1.

The complex number $a + ib$ is denoted by z. Given that $|z| = 4$ and $\arg z = \frac{1}{3}\pi$, find a and b. [4]

(Total 4 marks)

2.

One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3i$. Find the values of a and b. **[4]**

(Total 4 marks)

3.

The loci C_1 and C_2 are given by

$$|z| = |z - 4i| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + iy$, the complex number represented by the point of intersection of C_1 and C_2. [3]

(Total 8 marks)

4.

The complex numbers z and w are given by $z = 5 - 2i$ and $w = 3 + 7i$. Giving your answers in the form $x + iy$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

(Total 4 marks)

5.

Use an algebraic method to find the square roots of the complex number $21 - 20i$. [6]

(Total 6 marks)

6.

The roots of the equation

$$x^3 - 8x^2 + 28x - 32 = 0$$

are α, β and γ

Without solving the equation, find the value of

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(ii) $(\alpha + 2)(\beta + 2)(\gamma + 2)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(8)

(Total 8 marks)

7.

The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

(Total 7 marks)

8.

Please draw axes from -5 to 5.

(a) Sketch on the Argand diagram below, the locus of points satisfying the equation $|z - 2| = 2$

[2 marks]

(b) Given that $|z - 2| = 2$ and $\arg(z - 2) = -\frac{\pi}{3}$, express z in the form $a + bi$, where a and b are real numbers.

[3 marks]

(Total 5 marks)