

## Topic Y1 Complex numbers and roots of equations (Pre-TT B) [46] MS

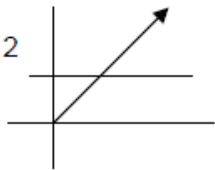
1.

<p><i>EITHER</i>  <math>a = 2</math></p> <p><math>b = 2\sqrt{3},</math>  <i>OR</i></p> <p><math>a = 2 \quad b = 2\sqrt{3}</math></p>	<p>M1  A1  M1  A1  M1  M1  A1 A1</p>	<p>4</p> <p>4</p>	<p>Use trig to find an expression for <math>a</math> (or <math>b</math>)  Obtain correct answer  Attempt to find other value  Obtain correct answer a.e.f.  (Allow 3.46 )  State 2 equations for <math>a</math> and <math>b</math></p> <p>Attempt to solve these equations  Obtain correct answers a.e.f.  SR <math>\pm</math> scores A1 only</p>
--	--	-------------------	---

2.

<p><math>x^2 - y^2 = 11</math> and <math>xy = 6\sqrt{5}</math></p> <p><math>\pm(2\sqrt{5} + 3i)</math></p>	<p>M1  A1  M1*  DM1  A1  A1  [6]</p>	<p>Attempt to equate real and imaginary parts of <math>(x + iy)^2</math> and <math>11 + 12\sqrt{5}</math>  Obtain both results cao  Obtain a quadratic in <math>x^2</math> or <math>y^2</math>  Solve a 3 term quadratic to obtain a value for <math>x</math> or <math>y</math>  Obtain 1 correct answer as complex number  Obtain only the other correct answer</p>
--	--	--

3.

<p>(i)</p>  <p>(ii)</p> <p><math>2\sqrt{3} + 2i</math></p>	<p>B1  B1  B1  B1  B1</p> <p>B1  M1  A1</p>	<p>5</p> <p>3</p> <p>8</p>	<p>Horizontal straight line in 2 quadrants  Through (0, 2)  Straight line  Through <math>O</math> with positive slope  In 1<sup>st</sup> quadrant only</p> <p>State or obtain algebraically that <math>y = 2</math>  Use suitable trigonometry  Obtain correct answer a.e.f. decimals OK must be a complex number</p>
--	---	----------------------------	---

4.

<p>(i) <math>11 - 29i</math></p>	<p>B1 B1</p>	<p>2</p>	<p>Correct real and imaginary parts</p>
<p>(ii) <math>1 + 41i</math></p>	<p>B1 B1</p>	<p>2</p> <p>4</p>	<p>Correct real and imaginary parts</p>

5.

<p><math>x^2 - y^2 = 21</math> and <math>xy = -10</math></p> <p><math>\pm(5 - 2i)</math></p>	<p>M1  A1A1  M1  M1  A1</p>	<p>6</p> <p>6</p>	<p>Attempt to equate real and imaginary parts of <math>(x + iy)^2</math> and <math>21 - 20i</math>  Obtain each result  Eliminate to obtain a quadratic in <math>x^2</math> or <math>y^2</math>  Solve to obtain <math>x = (\pm) 5</math> or <math>y = (\pm) 2</math></p> <p>Obtain correct answers as complex numbers</p>
--	---	-------------------	--

6.

(i)	$\alpha + \beta + \gamma = 8, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 28, \quad \alpha\beta\gamma = 32$	B1	3.1a
	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$= \frac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha+2)(\beta+2)(\gamma+2) = (\alpha\beta + 2\alpha + 2\beta + 4)(\gamma+2)$	M1	1.1b
	$= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) + 8$	A1	1.1b
	$= 32 + 2(28) + 4(8) + 8 = 128$	A1	1.1b
		(3)	
	<b>Alternative:</b>		
	$(x-2)^3 - 8(x-2)^2 + 28(x-2) - 32 = 0$	M1	1.1b
	$= \dots - 8 + \dots - 32 + \dots - 56 - 32 = -128$	A1	1.1b
	$\therefore (\alpha+2)(\beta+2)(\gamma+2) = 128$	A1	1.1b
	(3)		
(iii)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$	M1	3.1a
	$= 8^2 - 2(28) = 8$	A1ft	1.1b
		(2)	
<b>(8 marks)</b>			

7.

$\alpha + \beta = -k$

B1 State or use correct value

$\alpha\beta = 2k$

B1 State or use correct value

M1 Attempt to express sum of new roots in terms of  $\alpha + \beta, \alpha\beta$ 

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

A1 Obtain correct expression

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{2}(k-4)$

A1 Obtain correct answer a.e.f.

$\alpha'\beta' = 1$

B1 Correct product of new roots seen

$x^2 - \frac{1}{2}(k-4)x + 1 = 0$

B1ft Obtain correct answer, must be an eqn.

 $\boxed{7}$ 

Alternative for last 5 marks

M1 Obtain expression for  $u = \frac{\alpha}{\beta}$  in terms of  $k$  and $\alpha$  or  $k$  and  $\beta$ 

A1 Obtain a correct expression

A1 rearrange to get  $\alpha$  in terms of  $u$ 

M1 Substitute into given equation

A1 Obtain correct answer

8.

(a)	Draws 'circle' with centre $2 + 0i$ Ignore other features	AO1.1a	M1	
	Draws circle passing through $(0, 0)$ , $(4, 0)$ , close to $(2, 2)$ and $(2, -2)$ with Imaginary axis tangential	AO1.1b	A1	
(b)	Uses mod/arg forms	AO3.1a	M1	$z - 2 = 2 \left[ \cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right]$
	Substitutes exact values for cos and sin Allow one slip	AO1.1a	M1	$= 2 \left[ \frac{1}{2} + i \left( -\frac{\sqrt{3}}{2} \right) \right]$
	Obtains result in exact form	AO1.1b	A1	$z = 3 - \sqrt{3}i$
<b>Total</b>			<b>5</b>	