

Topic Y2 Counting principles & probability distributions (Post-TT A) [58]

1.

The discrete random variable X has probability distribution given by

x	2	3	6	11
$P(X = x)$	$\frac{1}{50}$	a	$\frac{1}{25}$	b

The discrete random variable $Y = X^2$

Given that $E(Y) = 50.3$

(a) find the value of a and the value of b

(3)

(b) Find $P(9 - Y > 0)$

(2)

Independent observations $X_1, X_2, X_3, \dots, X_{120}$ of X are taken.

The random variable T represents the total number of these 120 observations that are even.

(c) Find

(i) $E(T)$

(ii) $\text{Var}(T)$

(2)

(Total 7 marks)

2.

A variable X has the distribution $B(11, p)$.

(i) Given that $p = \frac{3}{4}$, find $P(X = 5)$.

[2]

(ii) Given that $P(X = 0) = 0.05$, find p .

[4]

(iii) Given that $\text{Var}(X) = 1.76$, find the two possible values of p .

[5]

(Total 11 marks)

3.

A committee of 7 people is to be chosen at random from 18 volunteers.

(i) In how many different ways can the committee be chosen?

[2]

The 18 volunteers consist of 5 people from Gloucester, 6 from Hereford and 7 from Worcester. The committee is to be chosen randomly. Find the probability that the committee will

(ii) consist of 2 people from Gloucester, 2 people from Hereford and 3 people from Worcester, [4]

(iii) include exactly 5 people from Worcester, [4]

[4]

(iv) include at least 2 people from each of the three cities. [4]

[4]

(Total 14 marks)

4.

A coin is biased so that the probability that it will show heads on any throw is $\frac{2}{3}$. The coin is thrown repeatedly.

The number of throws up to and including the first head is denoted by X . Find

- (i) $P(X = 4)$, [3]
- (ii) $P(X < 4)$, [3]
- (iii) $E(X)$. [2]

(Total 8 marks)

5.

An examination paper consists of 8 questions, of which one is on geometric distributions and one is on binomial distributions.

- (i) If the 8 questions are arranged in a random order, find the probability that the question on geometric distributions is next to the question on binomial distributions. [3]

Four of the questions, including the one on geometric distributions, are worth 7 marks each, and the remaining four questions, including the one on binomial distributions, are worth 9 marks each. The 7-mark questions are the first four questions on the paper, but are arranged in random order. The 9-mark questions are the last four questions, but are arranged in random order. Find the probability that

- (ii) the questions on geometric distributions and on binomial distributions are next to one another, [3]
- (iii) the questions on geometric distributions and on binomial distributions are separated by at least 2 other questions. [4]

(Total 10 marks)

6.

An office has a photocopier and a printer. The photocopier and the printer break down independently.

The number of breakdowns per month for the photocopier follows a Poisson distribution with mean 2.4

The number of breakdowns per month for the printer follows a Poisson distribution with mean 1.6

- (a) Determine which machine is more likely to break down exactly twice in one month. (2)
- (b) Work out the probability that both machines break down at most once in one month. (2)

In a randomly selected month there were 4 breakdowns.

- (c) Find the probability that in this month the photocopier broke down more than the printer. (4)

(Total 8 marks)