Topic Y2 Counting principles & probability distributions (Post-TT A) [58] MS

1.

Question	Scheme	Marks	AOs
2(a)	$[\mathrm{E}(Y)=]2^2\times\frac{1}{50} + 3^2\times a + 6^2\times\frac{1}{25} + 11^2\times b = 50.3$ $\frac{1}{50}+a+\frac{1}{25}+b=1$	M1	3.1a
	$\left.\begin{cases}9a+121b=48.78\\ a+b=0.94\end{cases}\right\}\rightarrow 112b=40.32$	M1	1.1b
	$\underline{a=0.58}$ and $\underline{b=0.36}$	A1	1.1b
		(3)	
(b)	P(9 – Y > 0) [= P(9 – X^2 > 0] = P(X < 3)	M1	1.1b
	= P(X = 2) $\underline{\frac{1}{50}}$	A1	1.1b
		(2)	
(c)(i)	$[\mathrm{E}(T)=120\times(\frac{1}{50}+\frac{1}{25})=]$ **<u>7.2</u>**	B1	1.1b
		(1)	
(ii)	$[\mathrm{Var}(T)=120\times(\frac{1}{50}+\frac{1}{25})\times(1-(\frac{1}{50}+\frac{1}{25}))=]$ awrt **<u>6.77</u>**	B1	1.1b
		(1)	
			(7 marks)

2.

9i	${}^{11}C_5$ x $(1/4)^6$ x $(3/4)^5$ 0.0268 (3 sfs)	M1 A1 2	or 462 x $(1/4)^6$ x $(3/4)^5$
ii	$q^{11}=0.05$ or $(1-p)^{11}=0.05$ $\sqrt[11]{0.05}$ $q=0.762$ or 0.7616 . . . $p=0.238$ (3 sfs)	M1 M1 A1 A1f 4	(any letter except p)$^{11}=0.05$ oe oe or invlog$(\frac{\log 0.05}{11})$ ft dep M2
iii	11 x p x $(1-p)=1.76$ oe $11p-11p^2=1.76$ or $p-p^2=0.16$ $11p^2-11p+1.76=0$ or $p^2-p+0.16=0$ $(25p^2-25p+4=0)$ $(5p-1)(5p-4)=0$ or $p=\frac{11-\sqrt{(11^2-4\text{x}11\text{x}1.76)}}{2\text{ x }11}$ $p=0.2$ or 0.8	M1 A1 A1 M1 A1 5	not $11pq=1.76$ any correct equn after mult out or equiv with = 0 or correct fact'n or subst'n for their quad equ'n eg $p=\frac{1\pm\sqrt{(1-4\text{x}0.16)}}{2}$
Total		**11**	

3.

(i) ${}^{18}C_7$ or ${}^{18!}/_{(11!\,x7!)}$ = 31824	M1 A1	2	cao
(ii) ${}^{5}C_2 \times {}^{6}C_2 \times {}^{7}C_3$ or 5250 ÷ 31824 = 875/5304 or 5250/31824 oe or 0.165 (3 sfs)	M2 M1 A1	4	M1: 1 correct ${}^{n}C_r$ or mult any three ${}^{n}C_r$s Divide by their (i). Indep If cancelled, must be clear have ÷ 31824
			5 x 4 x 6 x 5 x 7 x 6 x 5 x 7! 18x17x 16 x 15 x 14 x 13 x 12 x $2!^2$x3! Correct 7 fractions mult: M1 x7!: M1} ÷ ($2!^2$x3!): M1}both dep any 7 fracts mult
(iii) 5 from W & 2 from (G + H) ${}^{7}C_5 \times {}^{11}C_2$ or 1155 ÷ 31824 = 385/10608 or 1155/31824 oe or 0.0363 (3 sfs)	M1 M1 M1 A1	4	Seen or implied, eg by combs or list Divide by their (i). Indep
			7 x 6 x 5 x 4 x 3 x 11 x 10 x 7! 18x17x16 x 15 x14 x 13 x 12 x 5! x 2! Correct 7 fractions mult: M1 x 7! : M1} ÷ (5! x 2!): M1} both dep any 7 fracts mult
(iv) (2, 2, 3) or (2, 3, 2) or (3, 2, 2) ${}^{5}C_2\times{}^{6}C_2\times{}^{7}C_3+{}^{5}C_2\times{}^{6}C_3\times{}^{7}C_2$ $+{}^{5}C_3\times{}^{6}C_2\times{}^{7}C_2$ (÷ 31824) = 175/442 or 12600/31824 oe or 0.396 (3 sfs)	M1 M2 A1	4	Any one. Seen or implied eg by combs M1: one correct product. NOT ${}^{5}C_2\times{}^{6}C_2\times{}^{7}C_2$ (No mk for ÷ 31824)
			~~Equiv method: ((ii) ÷ etc) can imply M mks~~
			5 x 4 x 6 x 5 x 7 x 6 x 7! 18x17x16 x 15 x14 x 13 x $2!^2$x3! Correct 6 fractions mult: M1 x 7! : M1} ÷ ($2!^2$ x 3!) : M1}both dep any 6 fracts mult
			Complement method: Triple with total 7, incl at least one 0 or 1 or (0, 7) or (1, 6) seen or implied: M1 One correct prod seen, eg ${}^{5}C_0\text{x}{}^{6}C_2\text{x}{}^{7}C_5$ M1 Full correct method, incl "1 – " M1
	14		

4.

6i	Geo($^2/_3$) stated $(^1/_3)^3$ x $^2/_3$ = $^2/_{81}$ or 0.0247 (3 sfs)	M1 M1 A1 3	or implied by $(^1/_3)^n$ x $^2/_3$
ii	$(^1/_3)^3$ 1 - $(^1/_3)^3$ $^{26}/_{27}$ or 0.963 (3 sfs)	M1 M1 A1 3	or $^2/_3+{}^1/_3x^2/_3+(^1/_3)^2x^2/_3$: M2 one term omitted or extra or wrong: M1 1 - $(^1/_3)^4$ or 1- $(^2/_3+{}^1/_3x^2/_3+(^1/_3)^2x^2/_3$):M1
iii	1 / 2/3 = 3/2 oe	M1 A1 2	
Total		**8**	

5.

(i) $\frac{2\times 7!}{8!}$ $= ¼$	M1 M1 A1	3	7! and 8! used or ${}^{7}P_{7}$ and ${}^{8}P_{8}$ Correct formula, with "2 ×" Answer, $^1/_4$ or 0.25 only
(ii) $^1/_4$ or $4!\times 4!$ or $3!\times 3!$ or $^{3!}/_{4!}$ $\left(\frac{1}{4}\right)^2$ or $\frac{3!\times 3!}{4!\times 4!}$ $= {}^1/_{16}$	M1 M1 A1	3	Correct expression or 0.0625
(iii) Attempt subdivide, allow one error. Correct subdivision into 3 or 13 cases Correct expression $= \frac{13}{16}$	M1 M1 M1 A1	4	By description or listing or implied by probs, eg 1 – (ii) – P(sep by 1) All 3 or all 13 cases clearly present or 0.8125 or a.r.t. 0.813 only
Eg correct: $1-3\times\frac{1}{16}$; $1-(ii)-2\times\frac{3!\times 3!}{4!\times 4!}$ $\frac{3!\times 3!\times 13}{(4!\times 4!)}$; $(^3/_4)^2+2\times {}^1/_4\times {}^2/_4$			Eg incorrect: $1-\frac{3!\times 3!\times 3}{8!}$: M1M1M0A0 $1-{}^1/_{16}-\frac{3!\times 3!}{4!\times 4!}$: M1M0M0A0

6.

(a)	[X ~ Po(2.4) Y ~ Po(1.6)] $P(X=2)=\frac{e^{-2.4}\times 2.4^2}{2}=0.261\ldots$ $P(Y=2)=\frac{e^{-1.6}\times 1.6^2}{2}=0.258\ldots$	M1	1.1b
	Therefore the photocopier is more likely to break down exactly twice.	A1	1.1b
(b)	$P(X\leq 1)\times P(Y\leq 1)$	M1	1.1b
	[= 0.3084… × 0.5249…] awrt **0.162**	A1	1.1b
(c)	$\frac{P(X=4)\times P(Y=0)+P(X=3)\times P(Y=1)}{P(X+Y=4)}$	M1	3.1b
	$=\frac{\frac{e^{-2.4}2.4^4}{4!}\times e^{-1.6}+\frac{e^{-2.4}2.4^3}{3!}\times\frac{e^{-1.6}1.6}{1}}{\frac{e^{-4}4^4}{4!}}$	M1 M1	1.1b 1.1b
	$=\frac{297}{625}$ awrt **0.475**	A1	1.1b
			(8 marks)

Notes	
(a)	M1 Using each Poisson model to attempt each probability A1 Both correct awrt 3sf and correct conclusion
(b)	M1 Multiplication of two correct cumulative probabilities A1 awrt 0.162
(c)	1st M1 Correct ratio expression 2nd M1 Use of $P(X=4)\times P(Y=0)+P(X=3)\times P(Y=1)$ [0.125… × 0.201… + 0.209… × 0.323…] 3rd M1 Ratio of probabilities with denominator $\frac{e^{-4}4^4}{4!}$ [0.195…] A1 awrt 0.475