

Topic Y2 Counting principles & probability distributions (Post-TT B) [50]

1.

Each of the 7 letters in the word DIVIDED is printed on a separate card. The cards are arranged in a row.

- (i) How many different arrangements of the letters are possible? [3]
- (ii) In how many of these arrangements are all three Ds together? [2]

The 7 cards are now shuffled and 2 cards are selected at random, without replacement.

- (iii) Find the probability that at least one of these 2 cards has D printed on it. [3]

(Total 8 marks)

2.

- (i) A random variable X has the distribution $B(8, 0.55)$. Find

- (a) $P(X < 7)$, [1]
- (b) $P(X = 5)$, [2]
- (c) $P(3 \leq X < 6)$. [3]

- (ii) A random variable Y has the distribution $B(10, \frac{5}{12})$. Find

- (a) $P(Y = 2)$, [2]
- (b) $\text{Var}(Y)$. [1]

(Total 9 marks)

3.

The probability that a certain sample of radioactive material emits an alpha-particle in one unit of time is 0.14. In one unit of time no more than one alpha-particle can be emitted. The number of units of time up to and including the first in which an alpha-particle is emitted is denoted by T .

- (i) Find the value of

- (a) $P(T = 5)$, [3]
- (b) $P(T < 8)$. [3]

- (ii) State the value of $E(T)$. [2]

(Total 8 marks)

4.

A class consists of 7 students from Ashville and 8 from Bewton. A committee of 5 students is chosen at random from the class.

- (i) Find the probability that 2 students from Ashville and 3 from Bewton are chosen. [3]
- (ii) In fact 2 students from Ashville and 3 from Bewton are chosen. In order to watch a video, all 5 committee members sit in a row. In how many different orders can they sit if no two students from Bewton sit next to each other? [2]

(Total 5 marks)

5.

In a large region of derelict land, bricks are found scattered in the earth.

- (i) State two conditions needed for the number of bricks per cubic metre to be modelled by a Poisson distribution. [2]

Assume now that the number of bricks in 1 cubic metre of earth can be modelled by the distribution $Po(3)$.

- (ii) Find the probability that the number of bricks in 4 cubic metres of earth is between 8 and 14 inclusive. [3]
- (iii) Find the size of the largest volume of earth for which the probability that no bricks are found is at least 0.4. [4]

(Total 9 marks)

6.

The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X=x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geq -3) = 0.45$

- (a) find the probability distribution of X .

(7)

Given also that $E(Y^2) = 75$

- (b) find the exact value of $\text{Var}(X)$

(2)

- (c) Find $P(Y > X)$

(2)

(Total 11 marks)