

Topic Y2 Counting principles & probability distributions (Pre-TT A) [49] MS

1.

If P used instead of C consistently in all parts attempted (at least two parts attempted), max marks: (i) B0 (ii) M1A0 (iii) M1M1A0 Answers: (i) 427518000 (ii) 550368 (iii) 7338240			
i	593775	B1 [1]	or 594000 (3 sf)
ii	${}^{14}C_2 \times {}^9C_2 \times {}^7C_2$ alone = 68796	M1 A1 [2]	or 68800 (3 sf) MR: $+ {}^{30}C_6 (= \frac{84}{725} \text{ or } 0.116)$ M1A0
iii	14 (or ${}^{14}C_1$) \times ${}^{16}C_5$ or 14×4368 alone = 61152	M2 A1 [3]	or M1 for either ${}^{16}C_5$ or 4368 seen or 14 (or ${}^{14}C_1$) \times any no. seen or 61200 (3 sf) MR: $+ {}^{30}C_6 (= \frac{224}{2175} \text{ or } 0.103)$ M2A0 $14 \times ({}^9C_5 + {}^9C_4 \times 7 + {}^9C_3 \times 7 \times 2 + {}^9C_2 \times 7 \times 3 + 9 \times 7 \times 7 \times 5)$ M2 NOT 14 + : M0M0

2.

(i)	(a)	Geo(0.05) $(19/20)^5(1/20)$ = 0.0387	M1 M1 A1 3	Geo(0.05) or 0.95 stated or implied q^5p attempted Answer, a.r.t. 0.0387 ISW
	(b)	$(19/20)^{10}$ = 0.599	M1 M1 A1 3	q^{10} or $1 - p - pq \dots - pq^9$ [q^9 or q^{11} , or one wrong term: M1M0] Answer, a.r.t. 0.599 $1 - (19/20)^{10}$: M0M0A0
(ii)	Mean = $1/p$ = 20	M1 A1 2	20, cao	

3.

(a)	$P(X = 4) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}$	M1	2.1
	$= \frac{210}{720} = \frac{7}{24} *$	A1*	1.1b
(b)	$E(X) = 1 \times \frac{3}{10} + 2 \times \frac{7}{30} + 3 \times \frac{7}{40} + 4 \times \frac{7}{24} =$ $\frac{59}{24} = 2.458\bar{3}$ $E(X^2) = 1^2 \times \frac{3}{10} + 2^2 \times \frac{7}{30} + 3^2 \times \frac{7}{40} + 4^2 \times \frac{7}{24} =$ $\frac{299}{40} = 7.475$	M1	3.1b
	$\text{Var}(X) = \frac{299}{40} - \left(\frac{59}{24}\right)^2 = 1.43159 \dots$ awrt 1.43	M1 A1	1.1b 1.1b
(c)	Let rv $T =$ number of tokens drawn $T \sim \text{Geo}(0.3)$	B1	3.3
	$Y = T - 1$	B1	3.1b
	$\text{Var}(Y) = \text{Var}(T)$	M1	3.4
	$\text{Var}(Y) = \frac{1-p}{p^2} = \frac{0.7}{0.3^2} = \frac{70}{9} = 7.\bar{7}$	A1	1.1b

4.

(i)	$\frac{5!}{2}$ = 60	M1 A1 2	Allow 5P3
(ii)	4! = 24	M1 A1 2	Allow 2×4!
(iii)	$\frac{2!}{5} \times \frac{3!}{4}$ or $3/5 \times 2/4$ × 2 = $\frac{3}{5}$ oe	M1 M1 A1 3	allow M1 for $\frac{2!}{5} \times \frac{3!}{5} \times 2$ or $\frac{12}{25}$ or $(6 \times 3!) \div (i)$ M2 or $3! \div (i), 6 \div (i), (6+6) \div (i), 6k \div (i)$ or 6×6 or 36 or 1-correct answer M1 (k, integer ≤ 5)
Total		[7]	

5.

ia	0.299 (3 sf)	B1 1	
ib	0.2991 – 0.1040 = 0.195 (3 sf) or $\frac{1280}{6561}$ oe	M1 A1 2	Must subtract correct pair from table
iaa	${}^{15}C_4 \times (1-0.22)^{11} \times 0.22^4$ = 0.208 (3 sf)	M1 A1 2	Allow M1 for ${}^{15}C_4 \times 0.88^{11} \times 0.22^4$
iib	$(15 \times 0.22 =) 3.3$ $15 \times 0.22 \times (1-0.22)$ or $'3.3' \times (1-0.22)$ = 2.57 (3 sf)	B1 M1 A1 3	Allow M1 for $15 \times 0.22 \times 0.88$
Total		8	

6.

Question	Scheme	Marks	AOs
8(a)	$X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) = $P(X=2) \times P(Y=2) = 0.25104\dots \times 0.21685\dots$	M1	3.3
	= 0.05444... awrt <u>0.0544</u>	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$P(W=3) = 0.20458\dots$ awrt <u>0.205</u>	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2) \times 2)$	M1	3.3
	$P(T < 9) = 0.64819\dots$ awrt <u>0.648</u>	A1	1.1b
		(2)	
(d)	(i) Mean = $np = \underline{2.4}$	B1	1.1b
	(ii) Variance = $np(1-p) = 2.3904$ awrt <u>2.39</u>	B1	1.1b
		(2)	
(e)	(i) [$D \sim \text{Po}(2.4) \quad P(D \leq 4)$] = 0.9041... awrt <u>0.904</u>	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
(10 marks)			