Acceleration in horizontal circular motion

Starter

1. **(Review of last lesson)**
A cyclist completes a circuit of a circular track in 14 s. Calculate her exact angular speed.

Working: The cyclist travels through an angle of 2π in 14 s
So in 1 second she travels through $\frac{2\pi}{14} = \frac{\pi}{7}$ rad/s
Her exact angular speed is $\frac{\pi}{7}$ rad/s.

E.g. 1 A turntable is rotating at a constant rate of 45 rpm. A fly is standing on it, 8 cm from its centre. Find:
(a) the angular speed
(b) the tangential speed and
(c) the acceleration of the fly, stating the direction of the acceleration.

Working: (a) Angular speed, $\omega = \frac{45}{60} \times 2\pi = \frac{3\pi}{2}$ rad/s

(b) Tangential speed, $v = r\omega = 0.08 \times \frac{3\pi}{2} = \frac{3\pi}{25} \approx 0.377$ m/s

(c) Acceleration, $a_r = r\omega^2 = 0.08 \times \left(\frac{3\pi}{2}\right)^2 = \frac{9\pi^2}{50} \approx 1.78$ m/s².
The acceleration is directed towards the centre

E.g. 2 A particle of mass 0.4 kg moves on a horizontal circle, centre O. The speed of the particle is 3 m/s and the force on P directed towards O is 15 N. Calculate the distance OP.

Working: Using $F = ma$ radially: $15 = 0.4a_r \Rightarrow a_r = \frac{15}{0.44} = 37.5$
Using $a_r = \frac{v^2}{r}$: $37.5 = \frac{3^2}{r} \Rightarrow r = \frac{9}{37.5} = \frac{6}{25} = 0.24$
The distance OP is 0.24 m.

E.g. 3 A radial force of 20 N is required to maintain a particle moving in a horizontal circle of diameter 1.8 m with speed 4.8 m/s. Calculate the mass of the particle.

Working: $r = 0.9, v = 4.8$
Using $a_r = \frac{v^2}{r}$: $a_r = \frac{4.8^2}{0.9} = \frac{128}{5}$
Using $F = ma$ radially: $20 = m\frac{128}{5} \Rightarrow m = \frac{25}{32} = 0.78125$
The mass of the particle is 0.78125 kg

E.g. 4 A particle P of mass 0.3 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P moves in a circular path with speed 4 m/s.

(a) Calculate the tension in the string.

Given that the tension in the string cannot exceed 30 N, find

(b) the maximum tangential speed of P in m/s, and

(c) the maximum angular speed in rad/s.

Working:

(a) $r = 0.6, v = 4$

Using $a_r = \frac{v^2}{r}$: $\quad a_r = \frac{4^2}{0.6} = \frac{80}{3}$

Using $F = ma$ radially: $\quad F = 0.3 \times \frac{80}{3} = 8$

The tension in the string is 8 N.

(b) Using $F = ma$ radially: $\quad 30 = 0.3a_r \quad \Rightarrow \quad a_r = \frac{30}{0.3} = 100$

Using $a_r = \frac{v^2}{r}$: $\quad 100 = \frac{v^2}{0.6} \quad \Rightarrow \quad v^2 = 60$

$\therefore v = 2\sqrt{15} = 7.75$ (3 s.f.)

The maximum tangential speed of P is 7.75 m/s (3 s.f.)

(c) Using $v = r\omega$: $\quad 2\sqrt{15} = 0.6\omega$

$\omega = \frac{10\sqrt{15}}{3} \approx 12.9$ rad/s

The maximum angular speed is 12.9 rad/s (3 s.f.)

Video: Acceleration in horizontal circle

Solutions to Starter and E.g.s

Exercise

p87 4B Qu 1-6 (AS), 7-8 (A2 - needs coefficient of friction)