Average and Spread of Discrete Random Variables

Starter

1. (Review of last lesson)
The discrete random variable R has probability distribution function given by $P(R = r) = c(3 - r)$ for $r = 0, 1, 2, 3$. Find the value of the constant c.

Working: Since it is a random variable the sum of the probabilities add up to 1.

$$P(R = 0) + P(R = 1) + P(R = 2) + P(R = 3) = 1$$
$$c(3 - 0) + c(3 - 1) + c(3 - 2) + c(3 - 3) = 1$$
$$3c + 2c + c = 1$$
$$c = \frac{1}{6}$$

2. In the previous lesson we found the probability distribution, X, of a biased 4-sided dice was:

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

(a) If we rolled the dice 100 times, how many of each score would we expect to get?
(b) Using your expected frequencies from (a), find the mean score per roll.

Working: (a) Expected frequency = Probability × Number of trials
Expected frequency of $1 = P(X = 1) \times 100 = 0.48 \times 100 = 48$
The full table is:

x:	1	2	3	4
Expected frequencies:	48	24	16	12

(b) $$\text{Mean} = \frac{1 \times 48 + 2 \times 24 + 3 \times 16 + 4 \times 12}{100} = 1.92$$

E.g. 1 How could the answer to 2(b) be found from the probability distribution?

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

Working:

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

Mean $= 1 \times 0.48 + 2 \times 0.24 + 3 \times 0.16 + 4 \times 0.12 = 1.92$
The value 1.92 is called the mean of X or expected value of X.

E.g. 2 A random variable X has a pdf defined as shown. Find $E(X)$.

x:	-2	-1	0	1	2
$P(X = x)$:	0.3	0.1	0.15	0.4	0.05

Working:

x:	-2	-1	0	1	2
$P(X = x)$:	0.3	0.1	0.15	0.4	0.05

$$E(X) = (-2) \times 0.3 + (-1) \times 0.1 + 0 \times 0.15 + 1 \times 0.4 + 2 \times 0.05$$
$$= -0.2$$

E.g. 3 The random variable X has probability distribution as shown in the table.

x	1	2	3	4	5
$P(X=x)$	0.1	0.3	0.2	0.3	0.1

Find Var(X) and hence the standard deviation.

Working

$$\mu = E(X) = (1\times 0.1) + (2\times 0.3) + (3\times 0.2) + (4\times 0.3) + (5\times 0.1)$$
$E(X) = 3$ ***we could get this from the symmetry of the distribution***

$$E(X^2) = (1^2\times 0.1) + (2^2\times 0.3) + (3^2\times 0.2) + (4^2\times 0.3) + (5^2\times 0.1) = 10.4$$

$$\text{Var}(X) = E(X^2) - E^2(X) = 10.4 - 3^2 = 1.4$$

$$\text{Standard deviation} = \sqrt{1.4} = \frac{\sqrt{35}}{5} \approx 1.18$$

E.g. 4 Two fair cubical dice are rolled and S is the sum of their scores. Find:

(a) the distribution of S.

(b) the expected value of S

(c) the standard deviation of S

Working (a)

s:	2	3	4	5	6	7	8	9	10	11	12
$P(S=s)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(b)
$$E(S) = \left(2\times\frac{1}{36}\right) + \left(3\times\frac{2}{36}\right) + \left(4\times\frac{3}{36}\right) + \ldots + \left(12\times\frac{1}{36}\right)$$
$$E(S) = 7$$

(c)
$$E(S^2) = \left(2^2\times\frac{1}{36}\right) + \left(3^2\times\frac{2}{36}\right) + \ldots + \left(12^2\times\frac{1}{36}\right) = \frac{329}{6}$$

$$\text{Standard deviation} = \sqrt{\text{Var}(S)} = \sqrt{E(S^2) - E^2(S)} = \sqrt{\frac{329}{6} - 7^2} = \frac{\sqrt{210}}{6} \approx 2.42$$

Video: Expected values E(X)

Video: Variance Var(X)

Solutions to Starter and E.g.s

Exercise
p24 2A Qu 1i, 2-9, (10 red)