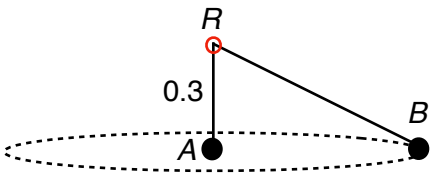
Banked tracks

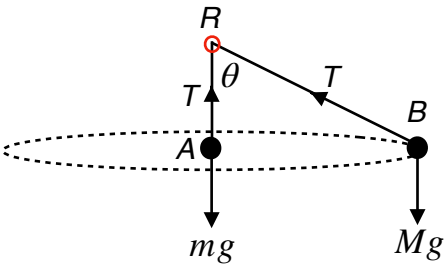
Starter

1. **(Review of last lesson)** A light inextensible string of length 0.8 m is threaded through a smooth ring, R, and carries a particle at each end. Particle A of mass m kg is at rest at a distance of 0.3 m below the ring. The other particle, B, of mass M kg is rotating in a horizontal circle whose centre is A.



(a) Express M in terms of m.

(b) Find the angular velocity of B.



Working:

(a) Since $AR = 0.3$, $RB = 0.5$.

By Pythagoras, $AB = 0.4 = r$

$$\sin\theta = 0.8$$
$$\cos\theta = 0.6$$
$$\tan\theta = \frac{4}{3}$$

$R(\uparrow)$ for A: $\quad T = mg$

$R(\uparrow)$ for B: $\quad T\cos\theta = Mg$

Replacing T by mg: $\quad 0.6mg = Mg$

$\therefore M = 0.6m$

(b) Using $F = ma$ radially: $\quad T\sin\theta = Ma_r$

But $T = mg$ and $M = 0.6m$: $\quad 0.8mg = 0.6ma_r$

$$a_r = \frac{4g}{3}$$

Using $a_r = r\omega^2$: $\quad \frac{4g}{3} = 0.4\omega^2$

$$\omega^2 = \frac{10g}{3}$$
$$\omega = \frac{7\sqrt{6}}{3} \approx 5.72$$

The angular velocity of B is 5.72 rad/s (3 s.f.)

E.g. 1 A car is rounding a bend of radius 100 m which is banked at an angle of $10°$ to the horizontal. At what speed must the car travel to ensure it has no tendency to slip sideways (i.e. design speed)?

Working:

$r = 100$

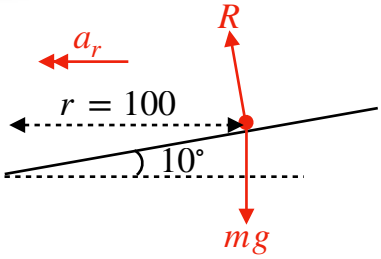
Using $a_r = \frac{v^2}{r}$: $\quad a_r = \frac{v^2}{100}$

$F = ma(\rightarrow)$: $\quad R\sin 10 = m\frac{v^2}{100}$

$R(\uparrow)$: $\quad R\cos 10 = mg$

Dividing:

$$\frac{\sin 10}{\cos 10} = \frac{v^2}{100g}$$
$$v^2 = 100g\tan 10$$
$$v \approx 13.1$$



The car must travel at 13.1 m/s (3 s.f.) for there to be no friction.

E.g. 2 A car is travelling around a circular bend on a road banked at an angle of α to the horizontal. The car may be modelled as a particle moving in a horizontal circle of radius 120 m. When the car is moving at a constant speed of 20 m/s there is no frictional force up or down the slope. Find the angle α, giving your answer to 1d.p.

Working: $r = 120, v = 20$

Using $a_r = \dfrac{v^2}{r}$: $\quad a_r = \dfrac{20^2}{120} = \dfrac{10}{3}$

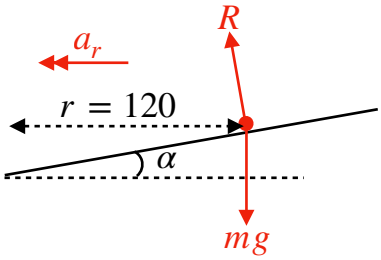
$F = ma(\rightarrow)$: $\quad R\sin\alpha = m\dfrac{10}{3}$

$R(\uparrow)$: $\quad R\cos\alpha = mg$

Dividing: $\quad \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{10}{3g}$

$$\tan\alpha = \frac{10}{3g}$$

$$\alpha = 18.8^\circ \text{ (3 s.f.)}$$



E.g. 3 A railway line is to be laid round a circular arc of radius 500 m. Trains are expected to travel at 45 km/h around the bend. Find:

(a) the force exerted by the outer edge on the flanges of the wheel if the track is level and the mass of the train is 35 tonnes

(b) the height, in mm, at which the outer rail should be raised so that there is no pressure on the wheel flanges, given that the rails are 1.5 m apart. Give your answer in centimetres.

Working: (a) $v = 45$ km/h = 12.5 m/s, $r = 500$

Using $a_r = \dfrac{v^2}{r}$: $\quad a_r = \dfrac{12.5^2}{500} = \dfrac{5}{16}$

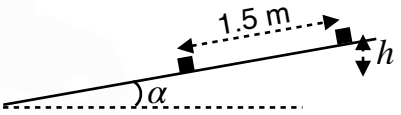
$F = ma(\rightarrow)$: $\quad F = 35000 \times \dfrac{5}{16}$

$$F = 10937.5 \text{ N} \approx 11 \text{ kN}$$

The force on the outer edge of the flanges of the wheel is 10937.5 N

(b) As before $v = 12.5, r = 500$

and $a_r = \dfrac{5}{16}$



$F = ma(\rightarrow)$: $\quad R\sin\alpha = 35000 \times \dfrac{5}{16}$

$R(\uparrow)$: $\quad R\cos\alpha = 35000g$

Dividing: $\quad \tan\alpha = \dfrac{5}{16g}$

$$\alpha \approx 1.826^\circ$$

Using trigonometry: $\quad \sin\alpha = \dfrac{h}{1.5}$

$$h = 1.5\sin 1.826^\circ$$

$$h = 0.0478 \text{ metres}$$

The height the outer rail should be raised is 4.78 cm

E.g. 4 A train of mass 80000 kg travels at 40 km/h round a bend of radius 1200 m.

(a) If the track is level, calculate the lateral thrust on the outer rail.

(b) At what height should the outer rail be raised to eliminate lateral thrust at this speed if the distance between the rails is 1.4 m. Give your answers to 2 s.f.

Working: (a) $v = 40$ km/h $= \frac{100}{9}$ m/s, $r = 1200$

Using $a_r = \frac{v^2}{r}$: $\quad a_r = \frac{100^2}{9^2 \times 1200} = \frac{25}{243}$

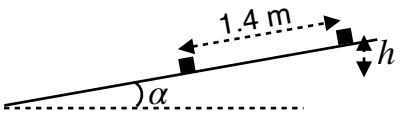
$F = ma(\rightarrow)$: $\quad F = 80000 \times \frac{25}{243}$

$F \approx 8230$ N

The force on the outer edge of the flanges is 8230 N (3 s.f.)

(b) As before $v = \frac{100}{9}$, $r = 1200$

and $a_r = \frac{25}{243}$



$F = ma(\rightarrow)$: $\quad R\sin\alpha = 80000 \times \frac{25}{243}$

$R(\uparrow)$: $\quad R\cos\alpha = 80000g$

Dividing: $\quad \tan\alpha = \frac{25}{243g}$

$\alpha \approx 0.601^\circ$

Using trigonometry: $\quad \sin\alpha = \frac{h}{1.4}$

$h = 1.4\sin 0.601^\circ$

$h = 0.0147$ metres

The height the outer rail should be raised is 1.5 cm

Video: Banked tracks

Solutions to Starter and E.g.s

Exercise

p96 4C Qu 4, 6, 9

Further questions on banked tracks

1. A car is moving at a constant speed of v m/s round a curve which is part of a horizontal circle of radius 50 m. The acceleration of the car is 3.38 m/s².
   (a) Calculate the value of v.
   (b) The car has a mass of 900 kg. Calculate the magnitude of the horizontal force acting on the car perpendicular to its direction of motion and suggest the physical cause of this force.

2. A curved section of race track, of radius 120 m, is banked at 40°.
   (a) Find the design speed.
   (b) Given that the car is of mass 840 kg and travels at 36 m/s, find the magnitude and direction of the lateral frictional force exerted by the track.

3. A road banked at 10° goes round a bend of radius, r m. Vehicles can travel at 11 m/s around the bend without tending to side-slip. Find the value of r.

Answers to further questions

1. (a) $v = 13$ (b) $900 \times 3.38 = 3042$ N, friction

2. (a) 31.4 m/s
   (b) Since 36 m/s $>$ design speed, car tries to slide out, so friction acts down the banked track.

3. The radius is about 70 m