

Combinations

Starter

1. (Review of previous material)

How many permutations of the letters of the word GROUP start with a vowel?

Working: The first letter must be a vowel so 2 ways
We are then arranging 4 letters i.e. 4!
Total ways = $2 \times 4! = 48$

2. (Review of last lesson)

Eight athletes run a 100m race. In how many ways can the first 3 places be filled?

Working: ${}^8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$

3. How many different ways are there to **choose** the letters A, B and C given that we can choose one, two or three letters?

N.B. The order you choose them in is not important.

Working: 3 ways to choose one letter
3 ways to choose two letters
1 way to choose three letters
So 7 ways in total

Special cases

- ${}^nC_r = \frac{n!}{(n-r)!r!} = \frac{n!}{r!(n-r)!} = {}^nC_{n-r}$

- What does $0!$ equal? Consider the question: "How many ways are there of choosing n objects from n available objects?"

$$1 \text{ way so } {}^nC_n = \frac{n!}{(n-n)!n!} = \frac{1}{0!} = 1 \text{ so } 0! = 1$$

A similar reasoning can be used for choosing no objects from n available objects.

- What does nC_1 mean in combination terms?

How many ways are there of choosing 1 object from n available objects?

$$n \text{ so } {}^nC_1 = n$$

- What does ${}^nC_{n-1}$ mean in combination terms?

How many ways are there of choosing $n - 1$ objects from n available objects?

$$n \text{ so } {}^nC_{n-1} = n$$

E.g. 1 A museum has places to hang 3 paintings from the five they have available. How many different **combinations** are there i.e. position is not important?

Working: Let the paintings be A, B, C, D and E.

If the question was a permutations problem, we would have ${}^5P_3 = \frac{5!}{2!}$

arrangements.

However, the actual arrangement of the strikers is unimportant (i.e. ABC = BAC).

So we have 3! times too many and need to divide by this number.

The number of combinations is $\frac{5!}{2!3!} = 10$

E.g. 2 The manager of a football team has a squad of 16 players. He needs to choose 11 to play in a match. How many teams can be chosen?

Working: ${}^{16}C_{11} = \frac{16!}{5! \times 11!} = 4368$

E.g. 3 In how many ways can 5 students be chosen from a class of 20 if the form captain must be chosen?

Working: Since the form captain must be chosen we have 4 picks from 19 students.

${}^{19}C_4 = \frac{19!}{15! \times 4!} = 3876$

E.g. 4 In how many ways can a party of 10 children be divided into 2 groups of:

- (a) 6 and 4 children
- (b) 5 children each

Working: (a) Each time a group of 6 is chosen, a group of 4 is chosen automatically
so ${}^{10}C_6 = 210$

(b) Each time a group of 5 is chosen, another group of 5 is formed automatically

i.e. ABCDE & FGHIJ is the same as FGHIJ & ABCDE

So rather than ${}^{10}C_5$ it is $\frac{1}{2} \times {}^{10}C_5 = 126$

E.g. 5 Five cards are dealt without replacement from a standard pack of 52 cards. Find the probability that exactly 3 of the 5 cards are hearts.

Working: The number of unrestricted choices is ${}^{52}C_5 = \frac{52!}{47! \times 5!}$
3 out of 13 hearts are chosen: ${}^{13}C_3 = \frac{13!}{10! \times 3!}$
2 out of 39 non-hearts are chosen: ${}^{39}C_2 = \frac{39!}{37! \times 2!}$
The number of 5 card sets with 3 hearts is ${}^{13}C_3 \times {}^{39}C_2$
Probability 3 of the 5 cards are hearts is $\frac{{}^{13}C_3 \times {}^{39}C_2}{{}^{52}C_5} = 0.0815$ (3 s.f.)

E.g. 6 Each question asks either for a number of combinations or for a number of permutations. Without attempting to find that number, decide whether you are looking for combinations or permutations.

- (a) How many arrangements of the letters X, Y and Z are there?
- (b) A team of 4 is to be chosen from nine players. How many different teams can be selected?
- (c) If 8 records can be taken to a desert island, from a collection of one hundred records, how many different sets can be chosen?
- (d) Five hundred raffle tickets are sold. When the 1st, 2nd and 3rd prizes are drawn, in how many ways can the prizes be won?
- (e) A museum lists its exhibits by code numbers of seven digits. How many code numbers are available?
- (f) A door is to be painted in two shades of green paint. If six suitable shades are available, in how many ways can the two shades be selected?
- (g) In how many ways can fifteen books be placed on a shelf?
- (h) A team has five strikers in the squad. The manager will choose two strikers to start the game and position is important.

Working:	(a) Permutation	(b) Combination
	(c) Combination	(d) Permutation
	(e) Permutation	(f) Combination
	(g) Permutation	(h) Permutation

[Video: Combinations](#)
[Video: Combining combinations](#)

[Solutions to Starter and E.g.s](#)

Exercise

p8 1C Qu 1i, 2i, 3-11, (12-14 red)