Components of acceleration (a general model) (A2)

Starter

1. (Review of last lesson) A particle is threaded on a smooth circular wire of radius, r. Starting from a point horizontally level with the centre, the particle needs an initial velocity of v to reach the highest point. Calculate the percentage increase in velocity needed if the particle starts from the lowest point of the circle.

Working: Horizontally level with centre: $\frac{1}{2}mv^2 = mgr \Rightarrow v = \sqrt{2gr}$

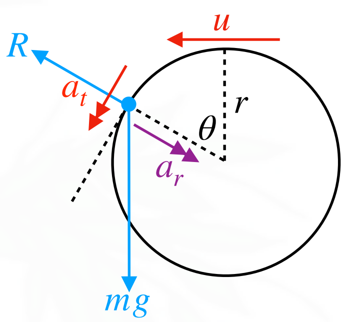
Lowest point: $\frac{1}{2}mv^2 = mg \times 2r \Rightarrow v = 2\sqrt{gr}$

Percentage increase $= \dfrac{2-\sqrt{2}}{\sqrt{2}} \times 100\,\% = 41.4\,\%$

E.g. 1 Consider a bead of mass m threaded on a smooth of wire in the shape of a circle of radius r. Initially the bead is at the highest point and has speed u. After the bead has rotated through an angle θ find, in terms of g, r and θ:

(a) the radial acceleration

(b) the tangential acceleration



Working: (a) $a_r = \dfrac{v^2}{r}$

New KE = Initial KE + Loss in GPE

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg(r - r\cos\theta)$$

$$v^2 = u^2 + 2gr(1-\cos\theta)$$

$$a_r = \frac{u^2 + 2gr(1-\cos\theta)}{r}$$

The radial acceleration is $\dfrac{u^2 + 2gr(1-\cos\theta)}{r}$

N.B. Using $F = ma$ radially would not work since we do not know R.

(b) Using $F = ma$ tangentially: $mg\sin\theta = ma_t$

$$a_t = g\sin\theta$$

The tangential acceleration is $g\sin\theta$

E.g. 2 A cyclist decreases her speed uniformly from 36 km/h to 27 km/h in 3 seconds while on the circular part of a horizontal track of radius 20 m. Find:

(a) the tangential acceleration
(b) an expression for the radial acceleration in terms of t
(c) the magnitude of the acceleration when $t = 1$.

Working: (a) 36 km/h $= 10$ m/s, 27 km/h $= 7.5$ m/s;

Tangential acceleration $= a_t = \dfrac{dv}{dt} = \dfrac{7.5 - 10}{3} = -\dfrac{5}{6}$

(b) By integrating, $v = 10 - \dfrac{5}{6}t$

So radial acceleration $= a_r = \dfrac{(10 - \frac{5}{6}t)^2}{20}$

(c) When $t = 1$, $a_r = 4.20$ m/s²

Magnitude of acceleration $= \sqrt{4.20\ldots^2 + \left(-\dfrac{5}{6}\right)^2} = 4.28$ m/s².

E.g. 3 A motorcyclist is rounding a bend of radius 20 metres. She enters the bend travelling at 10 m/s, and increase speed at a constant rate of 2 m/s² for each of the next 3 seconds. Find the magnitude of the acceleration and the angle it makes with the direction of motion at:

(a) the start of this time i.e. $t = 0$
(b) the end of this time i.e. $t = 3$

Working: (a) $a_t = 2$ so $v = \int 2dt = 2t + c$

When $t = 0$, $v = 10$ so $v = 2t + 10$

$a_r = \dfrac{(2t+10)^2}{20} = \dfrac{(t+5)^2}{5}$

When $t = 0, a_r = 5$ and $a_t = 2$

Magnitude of acceleration $= \sqrt{5^2 + 2^2} = \sqrt{29} = 5.39$ m/s² (3 s.f.)

Required angle $= \tan^{-1}\dfrac{5}{2} = 68.2°$ (3 s.f.)

(b) When $t = 3$, $a_r = 12.8$ and $a_t = 2$

Acceleration $= \sqrt{12.8^2 + 2^2} = \sqrt{29} = 13.0$ m/s² (3 s.f.)

Required angle $= \tan^{-1}\dfrac{12.8}{2} = 81.1°$ (3 s.f.)

Solutions to Starter and E.g.s

Exercise
p240 9B Qu 1-5 (red 6)