Conical pendulums

Starter

1. (Review of last lesson) A car, of mass 800 kg, is travelling at a steady speed of 15 m/s round a roundabout of radius 20 m on a flat road. Calculate:
   (a) the magnitude of the acceleration
   (b) the sideways force on each wheel assuming it is the same for each wheel.

Working: (a) Using $a_r = \dfrac{v^2}{r}$: $a_r = \dfrac{15^2}{20} = 11.25$ m/s².
The magnitude of the acceleration is 11.25 m/s².

(b) Using $F = ma$ radially: $F = 800 \times 11.25 = 9000$
The sideways force on each wheel is $\dfrac{9000}{4} = 2250$ N

N.B. In fact, the outside wheels go faster so the force is greater on the outside wheels.

2. (Review of last lesson) (A2 coefficient of friction). A small block A, of mass m kg, lies on a horizontal disc which is rotating about its centre B at 3 rad/s and $AB = 0.8$ m. If the block does not move relative to the disc, find the least value of coefficient of friction between the block and the disc.

Working: $\omega = 3,\ r = 0.8$

Using $a_r = r\omega^2$: $a_r = r\omega^2 = 0.8 \times 3^2 = \dfrac{36}{5}$

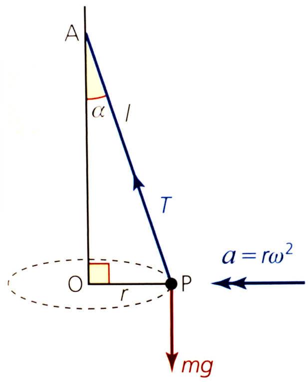
The radial force is provided by the friction and we'll need to use $F_{lim} = \mu R$.

Using $F = ma$ radially: $F = m \times \dfrac{36}{5} = \dfrac{36m}{5} = 7.2m$

$R(\uparrow)$: $R = mg$

Using $F_{lim} = \mu R$ $\quad \mu = \dfrac{7.2m}{mg} = 0.735$ (3 s.f.)

E.g. 1 A particle P, of mass m kg, is attached to a light inextensible string of length l m. The particle moves in a horizontal circle of radius r m. The string is fixed at A and makes an angle of α with the downward vertical. The tension in the string is T N.
The two forces acting on the particle are the weight and the tension in the string. By resolving vertically and using $F = ma$ horizontally, derive a formula for the angular speed, ω, in terms of g, l and α.



Working:

$R(\uparrow)$: $T\cos\alpha = mg$

$F = ma(\rightarrow)$: $T\sin\alpha = ma_r$

Since $a_r = r\omega^2$: $T\sin\alpha = mr\omega^2$

Using trigonometry: $r = l\sin\alpha$

$\therefore T\sin\alpha = m \times l\sin\alpha \times \omega^2$

$T = ml\omega^2$

Substituting into the first equation: $ml\omega^2\cos\alpha = mg$

$$\omega = \sqrt{\frac{g}{l\cos\alpha}}$$

E.g. 2 A particle is attached to one end of a light inextensible string of length 1.6 m. The other end of the string is attached to a fixed point. The particle moves, at constant speed, in a horizontal circle, with the string inclined at $55°$ to the vertical. Calculate the tangential speed of the particle.

Working: Use $F = ma$ radially: $T\sin 55 = ma_r$

$R(\uparrow)$: $T\cos 55 = mg$

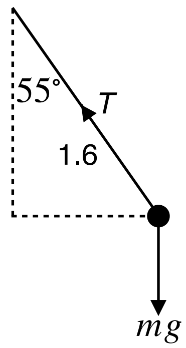
Dividing: $\dfrac{T\sin 55}{T\cos 55} = \dfrac{ma_r}{mg}$

$\therefore a_r = g\tan 55$ since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Replace a_r by $\dfrac{v^2}{r}$: $\dfrac{v^2}{1.6} = g\tan 55$

$v = \sqrt{1.6g\tan 55} \approx 4.28$ m/s

The tangential speed of the particle is 4.28 m/s (3 s.f.)



E.g. 3 In a simple model of a 'rotating swing', a particle of mass 30 kg is attached to one end of a light inextensible rope of length 2 m. The other end is attached to a fixed point O. The particle moves in a horizontal circle at a constant angular speed of 3 rad/s. The rope is inclined at a constant angle θ to the vertical. Find θ.

Working: $\omega = 3,\ r = 2\sin\theta$

Using $F = ma$ radially: $T\sin\theta = 30a_r$

Using $a_r = r\omega^2$: $a_r = 2\sin\theta \times 3^2 = 18\sin\theta$

Substitute into $T\sin\theta = 30a_r$: $T\sin\theta = 30 \times 18\sin\theta$

$T = 540$

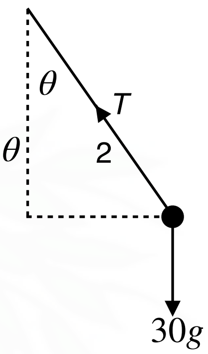
$R(\uparrow)$: $T\cos\theta = 30g$

$540\cos\theta = 30g$

$\cos\theta = \dfrac{g}{18}$

$\theta \approx 57.0°$

The value of θ is $57.0°$ (3 s.f.)



Video: Conical pendulum

Video: Motion on inside of cones

Video: Motion inside hemisphere

Solutions to Starter and E.g.s

Exercise

p96 4C Qu 1i, 2i, 3, 5, 7, 8