Counting Principles and Probability

Starter

1. **(Review of last lesson)**
Find the number of permutations of the letters of the word MISSISSIPPI.

Working: Permutations $= \dfrac{11!}{4!4!2!} = 34650$

E.g. 1 If the letters of the word MINIMUM are arranged in a line at random, what is the probability that the three M's are together at the beginning of the arrangement?

Working: 7 letters in total, 3 M's, 2 I's so total arrangements $= \dfrac{7!}{3!2!} = 420$

Ways of arranging the 4 non-M letters = $\dfrac{4!}{2!} = 12$

So probability = $\dfrac{12}{420} = \dfrac{1}{35}$

E.g. 2 Ten students are placed at random in a line. What is the probability that the two youngest pupils are separated?

Working: Total permutations $= 10!$

8 students means 9 gaps for the 2 youngest to choose: ${}^9C_2$

The 2 youngest can be arranged in $2!$ ways.

The other 8 students can be arranged in $8!$ ways

Total arrangements with youngest separated $= {}^9C_2 \times 2! \times 8! = 2903040$

Probability that the two youngest are separated $= \dfrac{{}^9C_2 \times 2! \times 8!}{10!} = \dfrac{4}{5}$

E.g. 3 From a group of 6 men and 8 women, five people are chosen at random. Find the probability that there are more men chosen than women.

Working: 5 men are chosen: ${}^6C_5$

4 men, 1 women chosen: ${}^5C_4 \times {}^8C_1$

3 men, 2 women chosen: ${}^5C_3 \times {}^8C_2$

Total combinations $= {}^{14}C_5$

Probability more men than women $= \dfrac{{}^6C_5 +{}^5C_4 \times {}^8C_1 +{}^5C_3 \times {}^8C_2}{{}^{14}C_5}$

$= \dfrac{49}{143}$

Video: Counting principles and probability

Solutions to Starter and E.g.s

Exercise

p17 1H Qu 1-5, (6 red)