

## Determinants of 2 by 2 Matrices

### Starter

1. Let  $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} -4 & 2 & 3 \\ -2 & 3 & -3 \end{pmatrix}$  and  $\mathbf{C} = (2 \ 3 \ 1)$ .

Calculate the following products if possible:

(a)  $\mathbf{AB}$                       (b)  $\mathbf{BA}$                       (d)  $\mathbf{CA}$                       (e)  $\mathbf{BC}$

**Working:** (a)  $\mathbf{AB}$     3 by 1  $\times$  2 by 3                       $\Rightarrow$     Not possible

(b)  $\mathbf{BA}$     2 by 3  $\times$  3 by 1                       $\Rightarrow$     Resultant matrix is 2 by 1

$$\begin{aligned} \mathbf{BA} &= \begin{pmatrix} -4 & 2 & 3 \\ -2 & 3 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} (-4) \times 1 + 2 \times (-1) + 3 \times 1 \\ (-2) \times 1 + 3 \times (-1) + (-3) \times 1 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ -8 \end{pmatrix} \end{aligned}$$

(d)  $\mathbf{CA}$     1 by 3  $\times$  3 by 1                       $\Rightarrow$     Resultant matrix is 1 by 1

$$\begin{aligned} \mathbf{CA} &= (2 \ 3 \ 1) \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \\ &= (2 \times 1 + 3 \times (-1) + 1 \times 1) \\ &= (0) \end{aligned}$$

Make sure your final answer is in brackets to show it is a matrix.

(e)  $\mathbf{BC}$     2 by 3  $\times$  1 by 3                       $\Rightarrow$     Not possible

**E.g. 1** Calculate the determinants for:                      (a)  $\mathbf{A} = \begin{pmatrix} 3 & -5 \\ 4 & 7 \end{pmatrix}$                       (b)  $\mathbf{B} = \begin{pmatrix} -6 & 2 \\ 4 & 9 \end{pmatrix}$

**Working:** (a)  $\det \mathbf{A} = \left| \begin{pmatrix} 3 & -5 \\ 4 & 7 \end{pmatrix} \right| = 3 \times 7 - 4 \times (-5) = 41$

(b)  $\det \mathbf{B} = \left| \begin{pmatrix} -6 & 2 \\ 4 & 9 \end{pmatrix} \right| = (-6) \times 9 - 4 \times 2 = -62$

**Video:**                      [Determinant of a 2 by 2 matrix](#)

[Solutions to Starter and E.g.s](#)

### Exercise

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