

Factorising Polynomials

Starter

1. (Review of last lesson)

Let $z_1 = \sqrt{3} - i$ and $z_2 = -\sqrt{2} + \sqrt{2}i$. By giving your answers $0 \leq \theta < 2\pi$, find:

(a) $\text{Arg}(z_2 \times z_1)$

(b) $\text{Arg}(z_2 \div z_1)$

(c) $\text{Arg}(z_1^2)$

Working: (a) $\text{Arg } z_1 = \text{Arg}(\sqrt{3} - i) = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$$\text{Arg } z_2 = \text{Arg}(-\sqrt{2} + \sqrt{2}i) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\text{Arg}(z_2 \times z_1) = \text{Arg } z_2 + \text{Arg } z_1 = \frac{3\pi}{4} + \frac{11\pi}{6} = \frac{31\pi}{12}$$

$\frac{31\pi}{12}$ is outside the range $0 \leq \theta < 2\pi$ so subtract 2π

$$\frac{31\pi}{12} - 2\pi = \frac{7\pi}{12}$$

$$\text{Arg}(z_2 \times z_1) = \frac{7\pi}{12}$$

(b) $\text{Arg}(z_2 \div z_1) = \text{Arg } z_2 - \text{Arg } z_1 = \frac{3\pi}{4} - \frac{11\pi}{6} = -\frac{13\pi}{12}$

$-\frac{13\pi}{12}$ is outside the range $0 \leq \theta < 2\pi$ so add 2π

$$-\frac{13\pi}{12} + 2\pi = \frac{11\pi}{12}$$

$$\text{Arg}(z_2 \div z_1) = \frac{11\pi}{12}$$

(c) $\text{Arg}(z_1^2) = 2 \times \frac{11\pi}{6} = \frac{11\pi}{3} = \frac{5\pi}{3}$

2. (Review of GCSE material) Factorise the quadratics:

(a) $x^2 - 4x - 12 = 0$

(b) $2x^2 + 7x - 15 = 0$

Working: (a) $(x - 6)(x + 2) = 0$

(b) $(2x - 3)(x + 5) = 0$

3. (Review of AS material) The polynomial $p(x) = x^3 + ax^2 + 2x + b$ is divisible by $x + 1$ and has a factor $x - 2$. Find the values of a and b .

Working: Divisible by $x + 1 \Rightarrow p(-1) = 0: -1 + a - 2 + b = 0$

Has a factor $x - 2 \Rightarrow p(2) = 0: 8 + 4a + 4 + b = 0$

Solve $a + b = 3$ and $4a + b = -12$ simultaneously

$a = -5, b = 8$

