# Finding dimensions from units and derivatives and predicting formulae

**Starter**

1. A formula is given by $t = kl^{\alpha}m^{\beta}g^{\gamma}$ where $k$ is dimensionless, $t$ is the time for $1$ period, $l$ is the length of string, $m$ is the mass at the end of the string and $g$ is the acceleration due to gravity. Use dimension analysis to determine the values of the powers $\alpha$, $\beta$ and $\gamma$.

***Working:*** $t = kl^{\alpha}m^{\beta}g^{\gamma} \Rightarrow \mathrm{T} = \mathrm{L}^{\alpha}\mathrm{M}^{\beta}(\mathrm{LT}^{-2})^{\gamma}$

Equating powers:
L: $0 = \alpha - 2\gamma$
M: $0 = \beta$
T: $1 = -2\gamma$

So $\alpha = \dfrac{1}{2}, \gamma = -\dfrac{1}{2}$ so $t = k\sqrt{\dfrac{l}{g}}$

***E.g. 1*** For any sphere the terminal velocity, $v$, is thought to depend on its radius, $r$, its weight, mg, and the viscosity of the liquid $\eta$ (in kg m$^{-1}$ s$^{-1}$).

(a) Write down a formula for $v$ as the product of unknown powers of $r$, $mg$, $\eta$ and a dimensionless constant $k$.

(b) Find the powers of $r$, $mg$ and $\eta$.

***Working:*** (a) $v = kr^{\alpha}(mg)^{\beta}\eta^{\gamma}$

(b) $[v] = \mathrm{LT}^{-1}$

$\left[kr^{\alpha}(mg)^{\beta}\eta^{\gamma}\right] = \mathrm{L}^{\alpha} \times (\mathrm{MLT}^{-2})^{\beta} \times (\mathrm{ML}^{-1}\mathrm{T}^{-1})^{\gamma}$

Equating powers:
L: $1 = \alpha + \beta - \gamma$
M: $0 = \beta + \gamma$
T: $-1 = -2\beta - \gamma$

Solving simultaneously: $\alpha = -1, \beta = 1, \gamma = -1$

So $V = \dfrac{kmg}{r\eta}$

***E.g. 2*** The rate of flow of the volume $R$ of a liquid with viscosity $\eta$ (in kg m$^{-1}$ s$^{-1}$) through a cylindrical pipe of length $l$ and an internal radius $r$ is believed to be of the form $R = k\eta^{w}l^{x}r^{y}p^{z}$ where $k$ is a dimensionless constant and $p$ (in Nm$^{-2}$) is the pressure difference between the ends of the pipe. Using dimension considerations, find:

(a) the values of $w$ and $z$

(b) the relationship between $x$ and $y$.

***Working:*** (a) $[R] = \mathrm{L}^{3}\mathrm{T}^{-1}$

$[k\eta^{w}l^{x}r^{y}p^{z}] = \left(\mathrm{ML}^{-1}\mathrm{T}^{-1}\right)^{w} \times \mathrm{L}^{x} \times \mathrm{L}^{y} \times \left(\mathrm{MLT}^{-2}\mathrm{L}^{-2}\right)^{z}$

Equating powers:
M: $w + z = 0$
T: $-w - 2z = -1$

So $w = -1$ and $z = 1$

(b) Equating powers: L: $3 = -w + x + y - z$

Since $w = -1$ and $z = 1$: $x + y = 3$

Video: Dimensional analysis

Video: Modelling using dimensional analysis

Solutions to Starter and E.g.s

**Exercise**
p41 2C Qu 1-10
p42 2D Qu 1 (table of dimensions)