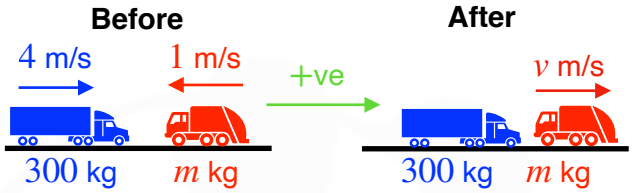
Impulsive tension in strings

Starter

1. **(Review of last lesson)** A truck of mass 300 kg travelling at 4 m/s collides with a second truck travelling at 1 m/s in the opposite direction. The first truck is brought to rest by the collision. If $e = 0.4$, find the mass of the second truck.

Working:



NLI: $0.4 \times (4 - -1) = v \quad \Rightarrow \quad v = 2$

CoM: $300 \times 4 - m = mv$

Substitute $v = 2$: $300 \times 4 - m = 2m \quad \Rightarrow \quad m = 400$

The mass of the second truck is 400 kg.

E.g. 1 Two particles A and B, joined by light inextensible string, are lying together on a smooth horizontal plane. The masses of A and B are 1 kg and 1.5 kg respectively. A is projected away from B with a speed of 5 m/s. Find

(a) the speed of each particle after the string jerks taut,

(b) the impulse in the string when it jerks taut.

Working:

(a) Momentum before $= 1 \times 5 = 5$

Momentum after $= (1 + 1.5)v$

$(1 + 1.5)v = 5$

$v = 2$ m/s

The speed of each particle after the string jerks taut is 2 m/s.

(b) ***Either*** Impulse = change in momentum for B

= final momentum − initial momentum

$= 1.5 \times 2 - 0 = 3$ Ns

or Impulse = change in momentum for A

= final momentum − initial momentum

$= 1 \times 2 - 1 \times 5 = -3$ Ns

N.B. The negative sign for A shows the impulse is acting in the opposite direction.

E.g. 2 Two particles A and B of equal mass m are connected by a light inextensible string of length l. Initially they are held at rest in the air, side by side. A is then released from rest.

(a) Find, in terms of l and g, A's speed just as the string is about to jerk taut.

(b) If B is released at this instant find, in terms of l and g, the common speed with which A and B together begins to move.

(c) Find the impulse in the string.

Working:

(a) A falls freely under gravity

Taking down as positive: $u = 0, s = l, a = 9.8, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 0^2 + 2gl$

$$v = \sqrt{2gl}$$

A's speed just as the string is about to jerk taut is $\sqrt{2gl}$

(b) Let the velocities of A and B when the string goes taut be V

Conservation of momentum: $m\sqrt{2gl} = mV + mV$

$$V = \frac{1}{2}\sqrt{2gl}$$

The common speed is $\frac{1}{2}\sqrt{2gl}$.

(c) ***Either*** Impulse = change in momentum for B

= final momentum − initial momentum

$$= -m \times \frac{1}{2}\sqrt{2gl}$$

or Impulse = change in momentum for A

= final momentum − initial momentum

$$= m\sqrt{2gl} - m \times \frac{1}{2}\sqrt{2gl}$$

$$= \frac{1}{2}m\sqrt{2gl}$$

The impulse in the string is $\frac{1}{2}m\sqrt{2gl}$ Ns

E.g. 3 Two particles, A and B, of masses 1.5 kg and 2.5 kg respectively are attached by a light, slack in extensible string and lie on a smooth horizontal surface. Particle A is initially at rest while particle B is projected at a speed of $4\sqrt{2}$ m/s and when the string becomes taut it makes an angle of $45°$ with the string. Find:

(a) the speed of A just after the string is taut

(b) the speed and direction of B just after the string is taut

(c) the impulse in the string

Working:

(a) CoM parallel to string: $2.5 \times 4\sqrt{2}\cos 45 = (1.5 + 2.5)v$

Velocity of A is $v = 2.5$ m/s

The speed of A just after the string is taut is 2.5 m/s.

(b) For B: Velocity parallel to string is 2.5 m/s

Velocity perpendicular to string is the same as before the string became taut i.e. $4\sqrt{2}\sin 45 = 4$

So velocity of B is $\sqrt{4^2 + 2.5^2} = 4.717$ m/s

Direction is $\tan^{-1}\dfrac{4}{2.5} = 58.0°$ to line of the string

(c) Impulsive tension for A = $1.5 \times 2.5 - 0 = 3.75$ Ns

E.g. 4 A light inextensible string has particles A and B, each of mass 400 g, attached to either end. The string and particles rest on a smooth table. The particle at A is hit with an impulse of 8 Ns at an angle $30°$ to the string, which is straight but not taut and at rest. What is the velocity of B after the impulse at the instant the string becomes taut?

Working:

Let u be the initial speed of A and v be the speed of A and B when the string becomes taut.

For A: Impulse = change in momentum: $0.4u = 8$

$u = 20$

CoM parallel to string: $0.4 \times 20\cos 30 = (0.4 + 0.4)v$

$v = 5\sqrt{3}$ m/s

The speed of B just after the string is taut is $5\sqrt{3}$ m/s along BA

N.B. Each question asks for the velocity of the particles immediately after the string becomes taut. This is because in most real-life cases the string will become slack again once the particles start moving.

Video: Impulsive tension example

Solutions to Starter and E.g.s

Exercise

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