Introduction to the Poisson Distribution

Starter

1. **(Review of last lesson)** In order to start a board game each player must throw at least one 6 with a pair of dice. Find the probability that for Jane to start she needs:
   (a) one throw
   (a) five throws
   (a) more than eight throws.

Working:

(a) The probability of success means either rolling a 6 and not a 6, ***or*** not a 6 and a 6 ***or*** or two 6s.

i.e. $p = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{11}{36}$

Alternatively: do the complementary event $p = 1 - \frac{5}{6} \times \frac{5}{6}$

(b) Let $X \sim \text{Geo}\left(\frac{11}{36}\right)$

$P(X = 5) = \left(\frac{25}{36}\right)^4 \times \frac{11}{36} \approx 0.0711$

The probability that five throws are needed is 0.0711 (3 s.f.)

(c) $P(X > 8) = P(X \geq 9) = \left(\frac{25}{36}\right)^8 \approx 0.0541$

The probability more than eight throws are needed is 0.0541 (3 s.f.)

E.g. 1 Discuss with your partner which of the following could be modelled by a Poisson distribution. Give reasons for your answers.

(a) The number of misprints on this page in the first draft of this book.

(b) The number of pigs in a given square metre of a field 1 hour after feed was placed in a central trough.

(c) The number of pigs in a given square metre of a field 1 minute after feed was placed in a central trough.

(d) The amount of salt, in mg, contained in 1 cm^3 of water taken from a bucket immediately after a teaspoon of salt was added.

(e) The number of marathon runners passing the finishing post between 20 and 21 minutes after the winning of the race.

Working:

(a) Yes, if they occur randomly

(b) Yes, pigs will have eaten and randomly dispersed

(c) No, pigs will be clustering around the trough

(d) No, salt has not had chance to dissolve and diffuse

(e) Yes, but possible clustering

E.g. 1 Given that $X \sim \text{Po}(3.25)$, use the formula to calculate, to 4 s.f.:
(a) $P(X = 3)$ (b) $P(X \leq 2)$ (c) $P(X \geq 2)$
Check your answers with the special function on your calculator.

Working: (a) ***By formula:*** $P(X = 3) = \dfrac{e^{-3.25} \times 3.25^3}{3!} = 0.2218$

Calculator: Use 2: Poisson PD $P(X = 3) = 0.2218$

(b) ***By formula:***
$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= \frac{e^{-3.25} \times 3.25^0}{0!} + \frac{e^{-3.25} \times 3.25^1}{1!} + \frac{e^{-3.25} \times 3.25^2}{2!}$$
$$= 0.3696$$

Calculator: Use 3: Poisson CD $P(X \leq 2) = 0.3696$

(c) ***By formula:***
$$P(X \geq 2) = 1 - P(X \leq 1)$$
$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$
$$= 1 - \frac{e^{-3.25} \times 3.25^0}{0!} - \frac{e^{-3.25} \times 3.25^1}{1!}$$
$$= 0.8352$$

Calculator: Use 3: Poisson CD
$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 0.8352$*
*Find $P(X \leq 1)$ >> STO >> A >> Menu >> 1 >> 1 — "ALPHA" A

E.g. 2 If $Y \sim \text{Po}(4.5)$, use the formula to find:

(a) $P(Y = 2)$ (b) $P(Y \leq 1)$ (c) $P(Y > 4)$ (d) $P(2 \leq Y \leq 6)$

Check your answers with the special function on your calculator.
For (d), use the special function on your calculator.

Working: (a) ***By formula:*** $P(Y = 2) = \dfrac{e^{-4.5} \times 4.5^2}{2!} = 0.1125$

Calculator: Use 2: Poisson PD $P(Y = 2) = 0.1125$

(b) ***By formula:***
$$P(Y \leq 1) = P(Y = 0) + P(Y = 1)$$
$$= \frac{e^{-4.5} \times 4.5^0}{0!} + \frac{e^{-4.5} \times 4.5^1}{1!}$$
$$= 0.0611$$

Calculator: Use 3: Poisson CD $P(Y \leq 1) = 0.0611$

(c) ***By formula:***
$$P(Y > 4) = 1 - P(Y \leq 4)$$
$$= 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)$$
$$= 1 - \frac{e^{-4.5} \times 4.5^0}{0!} - \frac{e^{-4.5} \times 4.5^1}{1!} - \frac{e^{-4.5} \times 4.5^2}{2!} - \frac{e^{-4.5} \times 4.5^3}{3!} - \frac{e^{-4.5} \times 4.5^4}{4!}$$
$$= 0.4679$$

Calculator: Use 3: Poisson CD
$P(Y > 4) = 1 - P(Y \leq 4) = 0.4679$*
*Find $P(Y \leq 4)$ >> STO >> A >> Menu >> 1 >> 1 — "ALPHA" A

(d) ***Calculator:*** Use 3: Poisson CD
$$P(2 \leq Y \leq 6) = P(Y \leq 6) - P(Y \leq 1)$$
$$= 0.83105 - 0.06110$$
$$= 0.7700$$

Video: Poisson distribution
Video: Poisson (changing the mean)
Video (Poisson PD): Finding Poisson PD with a calculator
Video (Poisson CD): Finding Poisson CD with a calculator
Video (list function): Using the list function

Solutions to Starter and E.g.s

Exercise
p41 3A 1ac, 2i