Keeping Objects Together or Separated

Starter

1. (Review of previous material) A team of 4 is chosen at random from 5 girls and 6 boys.
   (a) In how many ways can the team be chosen if
      (i) there are no restrictions;
      (ii) there must be more boys than girls
   (b) Find the probability that the team contains only 1 boy.

Working: (a) (i) ${}^{11}C_4 = 330$ ways

(ii) Team with only boys: ${}^{6}C_4 = 15$ ways
Team with 3 boys, 1 girl: ${}^{6}C_3 \times {}^{5}C_1 = 20 \times 5 = 100$
So total ways $= 115$

(b) Team with 1 boy, 3 girls: ${}^{6}C_1 \times {}^{5}C_3 = 6 \times 10 = 60$ ways
So probability $= \dfrac{60}{330} = \dfrac{2}{11}$

E.g. 1 Find the number of ways of arranging 6 women and 3 men to stand in a row so that all three men are standing together.

Working: Consider the 3 men as a single unit: they can be arranged in $3!$ ways
The 6 women and group of men constitute 7 units.
There are $7!$ ways of arranging 7 units.
Total ways $= 3! \times 7! = 30240$

E.g. 2 Nine different flower pots are to be arranged in a row. If the rose, daffodil, orchid and tulip pots must all be together, how many arrangements are possible?

Working: Consider the rose, daffodil, orchid and tulip (4 items) pots as one item.
There are now 6 items (5 pots plus 1 group) to arrange: $6!$
The rose, daffodil, orchid and tulip (4 items) pots can be arranged in $4!$ ways
Total arrangements $= 6! \times 4! = 17280$

E.g. 3 There are n objects that need to be arranged in a row. If r of them must all be together, where $r < n$, how many arrangement are possible?

Working: The r objects that must be together are considered 1 item
There are now $n - r + 1$ objects to arrange: $(n - r + 1)!$ ways
The r objects can be arranged in $r!$ ways
Total arrangements $= (n - r + 1)! \times r!$

Video: Permutations (items together)
Video: Permutations (items at ends)

E.g. 5 There are 3 boys and 4 girls in a team. The team photo has the players in a row and the boys must be separated.

(a) After removing the boys, how many gaps are there around and between the girls?
(b) How many ways can the 3 boys choose these gaps?
(c) How many ways can the 3 boys be arranged?
(d) How many ways can the 4 girls be arranged?
(e) What is the total number of arrangement of the team photo?

Working:

(a) 4 girls so 5 gaps (remember, there is one gap at each end)

(b) 3 boys are chosen for 5 gaps so ${}^5C_3$

(c) The 3 boys can be arranged in $3!$ ways

(d) The 4 girls can be arranged in $4!$ ways

(e) Total = ${}^5C_3 \times 3! \times 4! = 1440$ ways

E.g. 6 The word UNCOPYRIGHTABLE is one of two words of 15 letters with no repeated letters. How many permutations are of the letters of UNCOPYRIGHTABLE if the vowels must be separated?

Working:

10 consonants so 11 gaps
Choose 5 vowels for 11 gaps so ${}^{11}C_5$
Arrange the vowels = 5!
Arrange the consonants = 10!
Total = ${}^{11}C_5 \times 5! \times 10! \approx 2.012 \times 10^{11}$ ways

E.g. 7 A group of n distinct objects are to be arranged in a row but r of these objects must be separated. How many different arrangements are there?

Working:

$n - r$ objects left so $n - r + 1$ gaps: r objects for $n - r + 1$ gaps so ${}^{n-r+1}C_r$
The r objects are be arranged in $r!$ ways
The $n - r$ objects are be arranged in $(n - r)!$ ways
So total = ${}^{n-r+1}C_r \times r! \times (n - r)!$ ways

E.g. 8 Four girls and two boys, Alan and Tim are to sit in a row. In how many ways can the 6 children be arranged if:

(a) Alan and Tim must sit together
(b) Alan and Tim refuse to sit together

Working:

(a) Alan and Tim can be considered one unit so the arrangements of 5 objects is $5!$
However, for each arrangement, we could have Alan/Tim or Tim/Alan.
So arrangements $= 5! \times 2! = 240$

(b) Total arrangements $= 6!$
Arrangements where the boys sit together $= 5! \times 2! = 240$
Not sit together = Total arrangements − Sit together arrangements
$= 6! - 5! \times 2! = 480$

OR

There are 4 girls so there are 5 gaps to put the boys in (there are gaps on both ends of the girls).

The 4 girls can be arranged in $4!$ ways

The 2 boys can be arranged in $2!$ ways

The boys choose 2 gaps from a possible 5: ${}^5C_2$

Total = $4! \times 2! \times {}^5C_2 = 480$ ways

Video: Permutations (items separated)

Solutions to Starter and E.g.s

Exercise

p15 1F Qu 1-3, (4-5 red)