

## Matrix Multiplication

### Starter

1. For the matrix  $\mathbf{A} = \begin{pmatrix} 3 & 5 & -2 \\ 4 & -1 & -3 \\ 8 & 6 & -7 \end{pmatrix}$  state the number that corresponds to these elements:

(a)  $a_{21}$

(b)  $a_{32}$

(c)  $a_{13}$

**Working:** (a)  $a_{21}$  is the element in the **2nd** row, **1st** column  
 $a_{21} = 4$

(b)  $a_{32}$  is the element in the **3rd** row, **2nd** column  
 $a_{32} = 6$

(c)  $a_{13}$  is the element in the **1st** row, **3rd** column  
 $a_{13} = -2$

**E.g.** Find  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ .

**Working:**  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & \\ & \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} & af + bh \\ & \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ce + dg & \\ & \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} & cf + dh \\ & \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

**E.g. 1** Let  $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} -5 & 6 \\ 7 & 8 \end{pmatrix}$ . Find:

(a)  $\mathbf{AB}$

(b)  $\mathbf{BA}$

(c)  $\mathbf{A}^2$ .

**Working:** (a)  $\mathbf{AB} = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \times \begin{pmatrix} -5 & 6 \\ 7 & 8 \end{pmatrix}$   
 $= \begin{pmatrix} 1 \times (-5) + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ (-3) \times (-5) + 4 \times 7 & (-3) \times 6 + 4 \times 8 \end{pmatrix}$   
 $= \begin{pmatrix} 9 & 22 \\ 43 & 14 \end{pmatrix}$

$$\begin{aligned}
 \text{(b) } \mathbf{BA} &= \begin{pmatrix} -5 & 6 \\ 7 & 8 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} (-5) \times 1 + 6 \times (-3) & (-5) \times 2 + 6 \times 4 \\ 7 \times 1 + 8 \times (-3) & 7 \times 2 + 8 \times 4 \end{pmatrix} \\
 &= \begin{pmatrix} -23 & 14 \\ -17 & 46 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } \mathbf{A}^2 &= \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} 1 \times 1 + 2 \times (-3) & 1 \times 2 + 2 \times 4 \\ (-3) \times 1 + 4 \times (-3) & (-3) \times 2 + 4 \times 4 \end{pmatrix} \\
 &= \begin{pmatrix} -5 & 10 \\ -15 & 10 \end{pmatrix}
 \end{aligned}$$

**E.g. 2** Let  $\mathbf{A} = \begin{pmatrix} 2 & -3 & 1 \\ -5 & 2 & -2 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix}$ .

Decide whether it is possible to find these products:

- (a)  $\mathbf{AB}$                       (b)  $\mathbf{AC}$                       (c)  $\mathbf{BA}$                       (d)  $\mathbf{B}^2$

<b>Working:</b>	(a) $\mathbf{AB}$ : 2 by 3 $\times$ 3 by 2	Possible
	(b) $\mathbf{AC}$ : 2 by 3 $\times$ 2 by 2	Not possible
	(c) $\mathbf{BA}$ : 3 by 2 $\times$ 2 by 3	Possible
	(d) $\mathbf{B}^2$ : 3 by 2 $\times$ 3 by 2	Not possible

**E.g. 3** Using the matrices from **E.g. 2**, find  $\mathbf{AB}$ . Write down the dimensions of the resultant matrix.

**Working:**

$$\begin{aligned}
 \mathbf{AB} &= \begin{pmatrix} 2 & -3 & 1 \\ -5 & 2 & -2 \end{pmatrix} \times \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \\
 &= \begin{pmatrix} 2 \times 1 + (-3) \times 2 + 1 \times 3 & 2 \times 3 + (-3) \times 4 + 1 \times 6 \\ (-5) \times 1 + 2 \times 2 + (-2) \times 3 & (-5) \times 3 + 2 \times 4 + (-2) \times 6 \end{pmatrix} \\
 &= \begin{pmatrix} -1 & 0 \\ -7 & -19 \end{pmatrix}
 \end{aligned}$$

The dimensions of  $\mathbf{AB}$  are 2 by 2.

**E.g. 4** Let  $\mathbf{A} = \begin{pmatrix} 2 & -3 & 1 \\ -5 & 2 & -2 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix}$ .

State the dimensions of the resultant matrix of these products:

(a)  $\mathbf{BA}$  (b)  $\mathbf{CA}$

**Working:** (a)  $\mathbf{BA}$ : 3 by 2  $\times$  2 by 3 Resultant has dimensions 3 by 3

(b)  $\mathbf{CA}$ : 2 by 2  $\times$  2 by 3 Resultant has dimensions 2 by 3

**E.g. 5** Let  $\mathbf{P} = \begin{pmatrix} -1 & 2 & 3 \\ 4 & -5 & 6 \end{pmatrix}$  and  $\mathbf{Q} = \begin{pmatrix} 7 & 8 \\ 9 & -10 \\ 11 & 12 \end{pmatrix}$ . Find:

(a)  $\mathbf{PQ}$  (b)  $\mathbf{QP}$ .

**Working:** (a)  $\mathbf{PQ}$ : 2 by 3  $\times$  3 by 2  $\Rightarrow$  Resultant matrix is 2 by 2

$$\begin{aligned} \mathbf{PQ} &= \begin{pmatrix} -1 & 2 & 3 \\ 4 & -5 & 6 \end{pmatrix} \begin{pmatrix} 7 & 8 \\ 9 & -10 \\ 11 & 12 \end{pmatrix} \\ &= \begin{pmatrix} (-1) \times 7 + 2 \times 9 + 3 \times 11 & (-1) \times 8 + 2 \times (-10) + 3 \times 12 \\ 4 \times 7 + (-5) \times 9 + 6 \times 11 & 4 \times 8 + (-5) \times (-10) + 6 \times 12 \end{pmatrix} \\ &= \begin{pmatrix} 44 & 8 \\ 49 & 154 \end{pmatrix} \end{aligned}$$

(b)  $\mathbf{QP}$ : 3 by 2  $\times$  2 by 3  $\Rightarrow$  Resultant matrix is 3 by 3

$$\begin{aligned} \mathbf{QP} &= \begin{pmatrix} 7 & 8 \\ 9 & -10 \\ 11 & 12 \end{pmatrix} \begin{pmatrix} -1 & 2 & 3 \\ 4 & -5 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 7 \times (-1) + 8 \times 4 & 7 \times 2 + 8 \times (-5) & 7 \times 3 + 8 \times 6 \\ 9 \times (-1) + (-10) \times 4 & 9 \times 2 + (-10) \times (-5) & 9 \times 3 + (-10) \times 6 \\ 11 \times (-1) + 12 \times 4 & 11 \times 2 + 12 \times (-5) & 11 \times 3 + 12 \times 6 \end{pmatrix} \\ &= \begin{pmatrix} 25 & -26 & 69 \\ -49 & 68 & -33 \\ 37 & -38 & 105 \end{pmatrix} \end{aligned}$$

**Video:** [Matrix multiplication](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p12 1B Qu 1i, 2i, 3i, 4i, 5-12