

Matrix arithmetic

E.g. 1 Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$. Calculate the matrices:

(a) $\mathbf{A} + \mathbf{B}$ (b) $\mathbf{A} - \mathbf{B}$ (c) $3\mathbf{A} + 2\mathbf{B}$ (d) $4\mathbf{A} - 3\mathbf{B}$

Working: (a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 7 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & 5 \end{pmatrix}$

(c) $3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 4 & 6 \\ 8 & -2 \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 17 & 10 \end{pmatrix}$

(d) $4 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - 3 \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix} - \begin{pmatrix} 6 & 9 \\ 12 & -3 \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 0 & 19 \end{pmatrix}$

E.g. 2 Solve for \mathbf{X} the matrix equation $2\mathbf{X} + 3\mathbf{A} = 4\mathbf{X} - 3\mathbf{B}$. What do you need to assume for your answer to be correct?

Working:

$$2\mathbf{X} + 3\mathbf{A} = 4\mathbf{X} - 3\mathbf{B}$$

Add $3\mathbf{B} - 2\mathbf{X}$ to both sides: $3\mathbf{A} + 3\mathbf{B} = 2\mathbf{X}$

Divide both sides by 2: $\mathbf{X} = \frac{3}{2}(\mathbf{A} + \mathbf{B})$;

We need to assume that all matrices are the same dimensions.

E.g. 3 Write down the transpose of the following matrices:

(a) $\begin{pmatrix} 6 & -2 & 5 & 3 \\ 0 & 4 & -7 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 3 \\ -5 & 7 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}$

Working: (a) $\begin{pmatrix} 6 & -2 & 5 & 3 \\ 0 & 4 & -7 & 1 \end{pmatrix}^T = \begin{pmatrix} 6 & 0 \\ -2 & 4 \\ 5 & -7 \\ 3 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 8 & 3 \\ -5 & 7 \end{pmatrix}^T = \begin{pmatrix} 8 & 3 \\ -5 & 7 \end{pmatrix}$

(c) $\begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}^T = (2 \quad -7 \quad 3)$

N.B. A transposed column matrix becomes a row matrix, and vice versa.

E.g. 4 For the matrix $\mathbf{A} = \begin{pmatrix} 4 & 6 & -1 \\ 5 & 0 & -2 \\ 9 & 7 & -8 \end{pmatrix}$ state the number that corresponds to these elements:

(a) a_{13}

(b) a_{32}

(c) a_{23}

Working:

(a) a_{13} is the element in the **1st** row, **3rd** column
 $a_{13} = -1$

(b) a_{32} is the element in the **3rd** row, **2nd** column
 $a_{32} = 7$

(c) a_{23} is the element in the **2nd** row, **3rd** column
 $a_{23} = -2$

Video: [Dimensions of a matrix](#)

Video: [Matrix arithmetic](#)

[Solutions to Starter and E.g.s](#)

Exercise

p6 1A Qu 1i, 2i, 3i, 4i, 5i, 6i, 7-9