Mean and Variance of Binomial Distribution

Starter

1. **(Review of last lesson)**
    The random variable $W \sim B(10, 0.35)$. Use your calculator to find:
    (a) $P(W = 6)$ (b) $P(W \leq 6)$ (c) $P(W \geq 5)$ (c) $P(4 \leq W \leq 7)$

    Working: (a) $P(W = 6) = 0.0689$ **use Binomial PD**

    (b) $P(W \leq 6) = 0.9740$ **use Binomial CD**

    (c) $P(W \geq 5) = 1 - P(W \leq 4) = 0.2485$ **use Binomial CD**

    (d) $P(4 \leq W \leq 7) = P(W \leq 7) - P(W \leq 3) = 0.4814$
    use Binomial CD

E.g. 1 If the probability that it is a fine day is 0.4, find the expected number of fine days in a week, and the standard deviation. Explain the problem with the question.

   Working: $E(\text{fine days}) = np = 7 \times 0.4 = 2.8$ days
   Standard deviation $= \sigma = \sqrt{np(1 - p)} = \sqrt{7 \times 0.4 \times 0.6} = 1.30$ days
   Probability of weather one day in a week is not independent of weather on previous day

E.g. 2 The random variable X is such that $X \sim B(n, p)$, $E(X) = 2$ and $\text{Var}(X) = \dfrac{24}{13}$. Find:

   (a)   the values of n and p and
   (b) $P(X = 2)$.

   Working: (a) $E(X) = 2$ $\Rightarrow$ $np = 2$ $\therefore n = \dfrac{2}{p}$

   $\text{Var}(X) = \dfrac{24}{13}$ $\Rightarrow$ $np(1 - p) = \dfrac{24}{13}$

   $2(1 - p) = \dfrac{24}{13}$ $\Rightarrow$ $1 - p = \dfrac{12}{13}$ so $p = \dfrac{1}{13}$

   $\therefore n = \dfrac{2}{\frac{1}{13}} = 26$

   $n = 26$ and $p = \dfrac{1}{13}$

   (b) $P(X = 2) = {}^{26}C_2 \times \dfrac{1}{13}^2 \times \left(\dfrac{1}{13}\right)^{24}$

   $P(X = 2) = 0.282$

Video: Mean and variance of binomial

Solutions to Starter and E.g.s

Exercise
p30 2D Qu 1i, 2-6