Modulus-Argument Form of a Complex Number

Starter

1. **(Review of last lesson)**
Convert 240° to an angle in radians, expressing your answer in terms of π.

Working: $240^{\circ} \times \frac{\pi}{180} = \frac{4\pi}{3}$

2. **(Review of last lesson)** Find the modulus of $4 + 2\text{i}$.

Working: $\left|4 + 2\text{i}\right| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

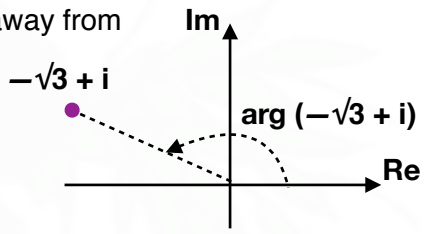
3. Calculate the argument of the complex numbers:
(a) $1 + \text{i}$ (b) 4i (c) $-\sqrt{3} + \text{i}$
Hint: use an Argand diagram to help you.

Working: (a) $\arg(1 + \text{i}) = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$

(b) $\arg 4\text{i} = \frac{\pi}{2}$ ***since 4i is on the positive imaginary axis ($y-$axis)***

(c) The acute angle needs to be taken away from π radians (180°)

$$\arg(-\sqrt{3} + \text{i}) = \pi - \tan^{-1}\frac{1}{\sqrt{3}}$$
$$= \frac{5\pi}{6}$$



4. You are given the modulus and argument of a complex number. Express the complex number in the form $x + y\text{i}$.
(a) Modulus = 6, argument = $\frac{\pi}{3}$ (b) Modulus = 2, argument = $\frac{3\pi}{2}$
Hint: draw an Argand diagram to help.

Working: (a) $x = 6\cos\frac{\pi}{6} = 3$
$y = 3\sin\frac{\pi}{6} = 3\sqrt{3}$
So complex number is $3 + 3\sqrt{3}\text{i}$

(b) $\frac{3\pi}{2} = 270^{\circ}$ so the complex number lies on the negative Imaginary axis
The complex number is -2i

E.g. 1 Convert $-1-\sqrt{3}\text{i}$ to $[r,\ \theta]$ form.

Working: $r = \left|-1-\sqrt{3}\text{i}\right| = \sqrt{(-1)^2+(-3)^2} = 2$

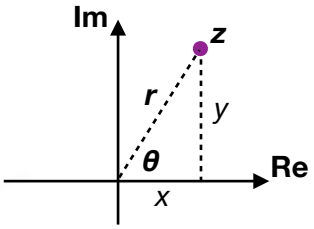
Ignore the signs: $\tan^{-1}\dfrac{\sqrt{3}}{1} = \dfrac{\pi}{3}$

$-1-\sqrt{3}\text{i}$ is in the 3rd quadrant so we need to add π (180^{o}) to the acute angle

$$\text{Arg}\,(-1-\sqrt{3}\text{i}) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$-1-\sqrt{3}\text{i} \equiv \left[2, \frac{4\pi}{3}\right]$$

E.g. 2 For the complex number $z = [r,\ \theta]$, express the $x-$ and $y-$ coordinates in terms of r and θ. Use the diagram to help you.



Working: By trigonometry,

$$\cos\theta = \frac{x}{r} \quad \Rightarrow \quad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \quad \Rightarrow \quad y = r\sin\theta$$

E.g. 3 Express the complex number $4\ \text{cis}\ \dfrac{\pi}{6}$ in Cartesian form.

Working: $4\ \text{cis}\ \dfrac{\pi}{6} \equiv 4\left(\cos\dfrac{\pi}{6} + \text{i}\sin\dfrac{\pi}{6}\right) = 2\sqrt{3} + 2\text{i}$

Video: Modulus-argument form of complex number

Solutions to Starter and E.g.s

Exercise

p127 4E Qu 1i, 2i, 3i, 4i, 5i, 6i, 7-11