Motion in a vertical circle (or conservation of mechanical energy)

Starter

1. **(Review of last lesson)** A circular banked race track has a radius of 200 m. Cars can go round the bend at 44 m/s without side-slipping. Find the angle at which the track is banked.

Working: $r = 200, v = 44$

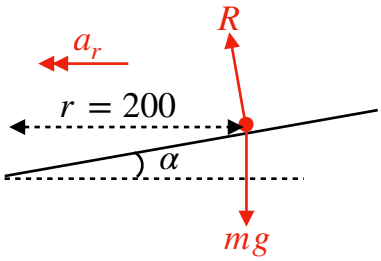
Using $a_r = \dfrac{v^2}{r}$: $\quad a_r = \dfrac{44^2}{200} = \dfrac{242}{25}$

$F = ma(\rightarrow)$: $\quad R\sin\alpha = m \times \dfrac{242}{25}$

$R(\uparrow)$: $\quad R\cos\alpha = mg$

Dividing: $\quad \tan\alpha = \dfrac{242}{25g}$

$\alpha = 44.6°$ (3 s.f.)



E.g. 1 A bead is projected with initial velocity 3 m/s from the lowest point of a smooth circular wire of radius 0.2 m fixed in a vertical plane. Find the speed of the bead when it has moved through:

(a) $180°$ (b) $60°$

Working: (a) Initial KE $= \dfrac{1}{2}m \times 3^2 = 4.5m$

GPE gain $= 0.4mg = 3.92m$

Final KE $= 4.5m - 3.92m = 0.58m$

$\therefore \dfrac{1}{2}mv^2 = 0.58m \quad \Rightarrow \quad v^2 = 1.16 \quad \Rightarrow \quad v \approx 1.08$

The speed of the bead is 1.08 m/s (3 s.f.)

(b) Initial KE $= \dfrac{1}{2}m \times 3^2 = 4.5m$

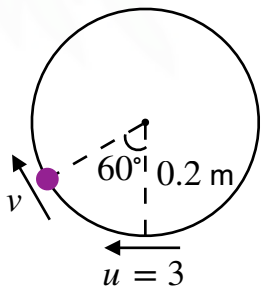
GPE gain $= 0.2mg(1 - \cos 60) = 0.98m$

Final KE $= 4.5m - 0.98m = 3.52m$

$\therefore \dfrac{1}{2}mv^2 = 3.52m$

$\Rightarrow \quad v^2 = 7.04 \quad \Rightarrow \quad v \approx 2.65$

The speed of the bead is 2.65 m/s (3 s.f.)



E.g. 2 A particle P is fixed to one end of a light rod of length 1.4 m. The other end is smoothly pivoted about O. The rod is released from rest when OP is horizontal. Find the speed of P when OP has rotated through:

(a) 90° (b) 60°.

Working: (a) GPE loss = KE gain $= 1.4mg$

$$\therefore \frac{1}{2}mv^2 = 1.4mg \quad \Rightarrow \quad v^2 = 2.8g \quad \Rightarrow \quad v \approx 5.24$$

The speed of P is 5.24 m/s (3 s.f.)

(b) GPE loss $= 1.4mg\sin 60$

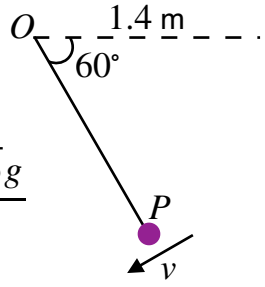
$$= \frac{7\sqrt{3}mg}{10}$$

KE gain = GPE loss

$$\therefore \frac{1}{2}mv^2 = \frac{7\sqrt{3}mg}{10} \quad \Rightarrow \quad v^2 = \frac{7\sqrt{3}g}{5}$$

$$\Rightarrow \quad v \approx 4.87$$

The speed of P is 4.87 m/s (3 s.f.)



E.g. 3 Decide whether the beads moving around these smooth vertical circles make a complete revolution. If so, find the velocity at the highest point of the circle. If not, find the height above the centre when it first comes to rest.

(a) Radius $= 1$ m, bead initially at lowest position, initial speed 8 m/s

(b) Radius $= 2$ m, bead initially level horizontally with centre, initial speed 6 m/s downwards

(c) Radius $= 0.5$ m, bead initially $\frac{\pi}{6}$ past the vertical, initial speed 1 m/s downwards

Working: (a) Initial KE $= \frac{1}{2}m \times 8^2 = 32m$

Required GPE gain $= mg \times 2 = 19.6m$

Since $32m > 19.6m$, the bead will reach the highest point.

KE at highest point $= 32m - 19.6m = 12.4m$

$$\therefore \frac{1}{2}mv^2 = 12.4m \quad \Rightarrow \quad v^2 = 24.8 \quad \Rightarrow \quad v \approx 4.98$$

The velocity at the highest point of the circle is 4.98 m/s (3 s.f.)

(b) Initial KE $= \frac{1}{2}m \times 6^2 = 18m$

Required GPE gain $= mg \times 2 = 19.6m$

Since $18m < 19.6m$, the bead will not reach the highest point.

Gain in GPE = Loss in KE $= 18m$

$$\therefore mgh = 18m \quad \Rightarrow \quad h = \frac{18}{g} \approx 1.84$$

The height above the centre when the bead first comes to rest is 1.84 m (3 s.f.).

(c) Initial KE $= \frac{1}{2}m \times 1^2 = \frac{1}{2}m$

Required GPE gain $= mg \times \left(0.5 - 0.5\cos\frac{\pi}{6}\right) \approx 0.656m$

Since $\frac{1}{2}m < 0.656m$, the bead will not reach the highest point.

GPE gain = KE loss: $mgh = \frac{1}{2}m$

$$h = \frac{5}{98} \approx 0.0510$$

Required height $= 0.5\cos\frac{\pi}{6} + \frac{5}{98} \approx 0.484$ (3 s.f.)

The height above the centre when the bead first comes to rest is 0.484 m.

Video: Motion in a vertical circle (string)

Video: Motion in a vertical circle (slack string)

Solutions to Starter and E.g.s

Exercise

p233 9A Qu 1i, 2i, 3-6 (red 7-9)