Permutations with Limited Places

Starter

1. **(Review of last lesson)** Consider the letters of the word PLYMOUTH.
   (a) In how many ways can the letters be arranged?
   (b) How many arrangements begin with two vowels?

   Working: (a) 8 different letters so $8!$

   (b) Arranging 2 vowels at the start: $2!$
   Arranging 6 consonants at the end: $6!$
   Total arrangements $= 2! \times 6! = 1440$

2. Twelve children stand in a row. In how many ways can they be arranged if one particular child has to stand at one end?

   Working: The child can stand at either end so 2 ways
   The other 11 children can be arranged in $11! = 39916800$ ways
   Total arrangements $= 2 \times 11! = 79833600$

3. A bookseller is organising her shop window and has 5 books to choose from. She only has room to display 3 books. How many arrangements can she make?

   Working: 1st position = 5 choices of book
   2nd position = 4 choices of book
   3rd position = 3 choices of book
   So $5 \times 4 \times 3 = 60$ ways
   Alternatively: there are 6 ways to ***arrange*** the letters ABC
   There are an additional 9 ways (total of 10 ways) to ***choose*** three letters:
   ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE
   So $6 \times 10 = 60$ ways

E.g. 1 From a group of 15 people, how many arrangements of 6 people can be made?

Working: $${}^{15}P_6 = \frac{15!}{(15-6)!} = \frac{15!}{9!} = 3603600$$

E.g. 2 In a goal of the month competition, there are 10 candidates. How many different arrangements for the top three positions are there?

Working: $${}^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$$

E.g. 3 Four different coloured disco lights are to be arranged in a vertical line. How many different arrangements can be made if there are seven different coloured lights to choose from?

Working: $${}^{7}P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$$

E.g. 4 How many five-digit numbers can be formed from 1, 2, 3, 4, 5, 6, 7, 8 and 9 if:

(a) the digits are all different

(b) the digits are all the same

(c) the digits are all different and the number is greater than 60000?

Working:

(a) ${}^9P_5 = \frac{9!}{4!} = 15120$ ways

(b) 11111, 22222, 33333 etc so 9 ways

(c) The first digit must be 6, 7, 8 or 9 so 4 ways.

Then there are 8 digits to arrange in 4 positions so ${}^8P_4$

$4 \times {}^8P_4 = 4 \times \frac{8!}{4!} = 6720$ ways

Video: Permutations (limited places)

Solutions to Starter and E.g.s

Exercise

p13 1E Qu 1-9, (10 red)