

## Permuting Objects with Repetitions

### Starter

1. (Review of last lesson)

In how many ways can 5 boys and 4 girls be arranged on a beach if:

- (a) boys and girls are in separate groups
- (b) John, Olivia and Michael want to stay together
- (c) the four girls must be separated
- (d) boys and girls alternate

- Working:**
- (a) Ways of arranging 5 boys:  $5!$   
 Ways of arranging 4 girls:  $4!$   
 Ways of arranging 2 groups (boys and girls):  $2!$   
 Total ways =  $5! \times 4! \times 2! = 5760$
  - (b) Consider John, Olivia and Michael as one group.  
 There are now 7 items to arrange:  $7!$  ways  
 John, Olivia and Michael can be arranged in  $3!$  ways  
 Total ways =  $7! \times 3! = 30240$
  - (c) 5 boys so 6 gaps for the 4 girls to choose from:  ${}^6C_4$  ways  
 The 4 girls can be arranged in  $4!$  ways  
 The 5 boys can be arranged in  $5!$  ways  
 Total ways =  ${}^6C_4 \times 4! \times 5! = 43200$
  - (d) The 5 boys can be arranged in  $5!$  ways  
 The 4 girls can be arranged in  $4!$  ways  
 Total ways =  $5! \times 4! = 2880$

**E.g. 1** Calculate the permutations of the letters of the word DIGIT.

- Working:** The number of permutations of D I<sub>1</sub> G I<sub>2</sub> T is  $5!$   
 But D I<sub>1</sub> G I<sub>2</sub> T  $\equiv$  D I<sub>2</sub> G I<sub>1</sub> T  
 I<sub>1</sub> and I<sub>2</sub> can be arranged in  $2!$  ways *these are identical so divide by 2!*  
 Total ways =  $\frac{5!}{2!} = 60$

**E.g. 2** (a) Calculate the arrangements of the letters of the word DEFEATED.  
 (b) If all the E's must be separated, how many permutations of the letters in the word DEFEATED are there?

- Working:**
- (a) 3 Es and 2 Ds so number of arrangements is  $\frac{8!}{2!3!} = 3360$
  - (b) 5 letters so 6 gaps for 3 E's to choose from so  ${}^6C_3$  ways  
 1 permutation of 3 E's  
 Arrangements of DFATD =  $\frac{5!}{2!}$   
 So total ways is  ${}^6C_3 \times 1 \times \frac{5!}{2!} = 1200$

**E.g. 3** Consider the letters of the word ADDING. How many permutations are there if:

- (a) there are no restrictions
- (b) the 2 D's together
- (c) the 2 D's separated

**Working:** (a) 6 letters but 2 are identical so  $\frac{6!}{2!} = 360$  arrangements

(b) With the 2 D's together there are now 5 items to arrange  
Total arrangements =  $5! = 120$

**N.B.** Since the D's are identical we do not multiply by 2!

(c) 4 non-D letters so 5 gaps for the 2 D's to choose from:  ${}^5C_2$  ways  
The 2 D's can be arranged in only 1 way  
The 4 non-D letters can be arranged in  $4!$  ways  
Total permutations =  ${}^5C_2 \times 1 \times 4! = 240$

**Video:** [Permutations with repeated items](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p16 1G Qu 1-7, (8-10 red)