Poisson Distribution (worded questions)

Starter

1. (Review of last lesson)

The random variable $X \sim \text{Po}(3.5)$, find the values of a, b, c and d such that:

(a) $P(X \leq a) = 0.8576$ (b) $P(X > b) = 0.6792$

(c) $P(X \leq c) \geq 0.95$ (d) $P(X > d) \leq 0.005$

Hint: Use trial and improvement with your calculator

Working:

(a) Use 3: Poisson CD
By trial and improvement, $a = 5$

(b) $P(X > b) = 0.6792 \quad \Rightarrow \quad P(X \leq b) = 1 - 0.6792 = 0.3208$
Use 3: Poisson CD
By trial and improvement, $b = 2$

(c) Use 3: Poisson CD
By trial and improvement, $c = 7$

(d) $P(X > d) \leq 0.005 \quad \Rightarrow \quad P(X \leq d) = 1 - 0.005 = 0.995$
Use 3: Poisson CD
By trial and improvement, $d = 9$

2. The number of demands for taxis to a taxi firm is Poisson distributed with, on average, 4 demands every 30 minutes. Find the probabilities, to 4 s.f., of:

(a) no demand in 30 minutes
(b) 1 demand in 1 hour
(c) fewer than 2 demands in 15 minutes

Working:

(a) For 30 minutes: $X \sim \text{Po}(4)$

Formula: $P(X = 0) = \dfrac{e^{-4} \times 4^0}{0!} \approx 0.0183$

Calculator: $P(X = 0) \approx 0.0183$

The probability there is no demand in 30 minutes is 0.0183 (4 s.f.)

(b) The length of time has doubled so for 1 hour consider $Y \sim \text{Po}(8)$

Formula: $P(Y = 1) = \dfrac{e^{-8} \times 8^1}{1!} \approx 0.0268$

Calculator: $P(Y = 1) \approx 0.0268$

The probability there is 1 demand in 1 hour is 0.0268 (4 s.f.)

(c) The length of time has halved so for 15 min consider $W \sim \text{Po}(2)$

Formula:
$$\begin{aligned} P(W < 2) &= P(W = 0) + P(W = 1) \\ &= \frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} \\ &= 0.1353... + 0.2706... \\ &\approx 0.4060 \end{aligned}$$

Calculator: Use 3: Poisson CD $P(W < 2) = P(W \leq 1) \approx 0.406$

The probability there is fewer than 2 demands in 15 minutes is 0.4060 (4 s.f.)

E.g. 1 A technician is responsible for a large number of machines. Minor adjustments have to be made to the machines and these occur at random and at a constant average rate of 7 per hour. Find, to 4 sf, the probability that:

(a) in a given hour the technician makes four or fewer adjustments,

(b) during a half-hour break no adjustments are required.

Working: (a) $X \sim \text{Po}(7)$

Formula:

$$P(X \leq 4) = P(X=0) + \ldots + P(X=4) = \frac{e^{-7} \times 7^0}{0!} + \ldots + \frac{e^{-7} \times 7^4}{4!} \approx 0.1730$$

Calculator: Use 3: Poisson CD $P(W \leq 4) \approx 0.1730$

The probability that in a given hour the technician makes four or fewer adjustments is 0.1730 (4 s.f.)

(b) Halve the time, halve the mean so $Y \sim \text{Po}(3.5)$

Formula:

$$P(Y=1) = \frac{e^{-3.5} \times 3.5^0}{0!} \approx 0.0302$$

Calculator: $P(Y=0) \approx 0.0302$

The probability that during a half-hour break no adjustments are required is 0.0302 (4 s.f.)

E.g. 2 During working hours an office switchboard receives telephone calls at random at an average of one call every 40 seconds.

(a) Find, to 3 decimal places, the probability that during a given one-minute period:

(i) no call is received,

(ii) at least 2 calls are received.

(b) Find, to 3 decimal places, the probability that no call is received between 10:30am and 10:31am and that at least two calls are received between 10:31am and 10:32am.

Working: (a) 1 call every 40 seconds ≡ 1.5 calls every minute so $X \sim \text{Po}(1.5)$:

(i) ***Formula:***

$$P(X=0) = \frac{e^{-1.5} \times 1.5^0}{0!} \approx 0.223$$

Calculator: Use 2: Poisson PD: $P(X=0) \approx 0.223$

The probability that no call is received is 0.223 (3 d.p.)

(ii) ***Formula:***

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X=0) - P(X=1) = 1 - \frac{e^{-1.5} \times 1.5^0}{0!} - \frac{e^{-1.5} \times 1.5^1}{1!} \approx 0.442$$

Calculator: Use 3: Poisson CD

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.5578\ldots \approx 0.442$$

The probability that at least 2 calls are received is 0.442 (3 d.p.)

(b) $P(X=0) \times P(X \geq 2) \approx 0.223 \times 0.442 \approx 0.099$

The required probability is 0.099 (3 d.p.)

E.g. 3 A shop sells a particular make of radio at a rate of 4 per week on average. The number sold in a week has a Poisson distribution.

(a) Find the probability that the shop sells at least 2 in a week.

(b) Find the smallest number that can be in stock at the beginning of the week in order to have at least a $99\,\%$ chance of being able to meet all demands during that week.

Give your answers to 4 s.f.

Working: (a) $X \sim \text{Po}(4)$

Formula:

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \frac{e^{-4} \times 4^0}{0!} - \frac{e^{-4} \times 4^1}{1!} \\ &\approx 0.9084 \end{aligned}$$

Calculator: Use 3: Poisson CD

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - 0.091578... \\ &\approx 0.9084 \end{aligned}$$

The probability that the shop sells at least 2 in a week is 0.9084 (3 d.p.)

(b) $P(X \leq x) > 0.99$

Use 3: Poisson CD and trial and improvement

$P(X \leq 8) \approx 0.979$ and $P(X \leq 9) \approx 0.992$

The smallest number that can be in stock is 9

Video: Finding an observed value
Video: Poisson distribution EQ

Solutions to Starter and E.g.s

Exercise

p41 3A 3, 4-6, 9-12, (7-8 A2, 13-15 red)