

Probability distributions (AS Ma)

Starter

1. Use your calculator to find the mean and standard deviation of the data set:
28, 41, 18, 24, 37, 52, 33, 35.

Working: Mean = 33.5
Standard deviation = 9.84 (3 s.f.)

E.g. 1 A tetrahedral die has the numbers 1, 2, 3 and 4 on its faces. The die is biased in such a way that the probability of the die landing on the number x is inversely proportional to x .

For example, $P(X = 3) = \frac{k}{3}$ where k is a constant.

- (a) Write down the probability density function in terms of k .
(b) Find the probability distribution for X , the number the die lands on after a single roll.

Working: (a) $P(X = x) = \frac{k}{x}$ where $x = 1, 2, 3, 4$
(b) The sum of the probabilities is 1 so $\frac{k}{1} + \frac{k}{2} + \frac{k}{3} + \frac{k}{4} = 1$
so $k = \frac{12}{25}$

x	1	2	3	4
$P(X = x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

E.g. 2 Let X be the discrete variable 'the number of fours obtained when two dices are thrown'.

- (a) Find the probability distribution.
(b) Show that X is random variable.

Working: (a) $P(X = 0) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$
 $P(X = 1) = 2 \times \frac{5}{6} \times \frac{1}{6} = \frac{10}{36} = \frac{5}{18}$
 $P(X = 2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
Probability distribution

x :	0	1	2
$P(X = x)$:	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

- (b) $\frac{25}{36} + \frac{10}{36} + \frac{1}{36} = 1$ so X is a random variable

[Video: Probability distribution tables](#)

[Solutions to Starter, E.g.s and Exercise](#)

Exercise

Answers can be found via the [blue link](#) above.

1. A discrete random variable X has the following probability distribution:

$$\begin{array}{cccc} x: & 1 & 2 & 3 & 4 \\ P(X = x): & \frac{1}{3} & \frac{1}{3} & k & \frac{1}{4} \end{array} \quad \text{where } k \text{ is a constant}$$

- (a) Find the value of k .
(b) Find $P(X \leq 3)$.
2. The probability density function of a discrete random variable is given by $P(X = x) = kx$ for $x = 12, 13, 14$. Find the value of the constant k .
3. The pdf of a discrete random variable Y is given by $P(Y = y) = cy^2$ for $y = 0, 1, 2, 3, 4$. Given that c is a constant find its value.
4. Two tetrahedral dice, each with faces labelled 1, 2, 3 and 4 are thrown and the score noted, where the score is the sum of the two numbers. If X is the random variable 'the score when two tetrahedral dice are thrown':
(a) Find the probability distribution of X .
(b) Find the probability density function of X .
5. A drawer contains 8 brown and 4 blue socks. A sock is taken from the drawer at random, its colour is noted and it is then replaced. The procedure is done three times. If X is the random variable 'the number of brown socks taken', find the probability distribution for X .

Answers to exercise

1. (a) $k = \frac{1}{12}$ (b) $\frac{3}{4}$

2. $k = \frac{1}{39}$

3. (a)
$$\begin{array}{cccccccc} x: & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ P(X = x): & \frac{1}{16} & \frac{2}{16} & \frac{3}{16} & \frac{4}{16} & \frac{3}{16} & \frac{2}{16} & \frac{1}{16} \end{array}$$

(b)
$$P(X = x) = \frac{x-1}{16} \text{ for } x = 2, 3, 4, 5$$
$$P(X = x) = \frac{9-x}{16} \text{ for } x = 6, 7, 8$$

5.
$$\begin{array}{cccc} x: & 0 & 1 & 2 & 3 \\ P(X = x): & \frac{1}{27} & \frac{6}{27} & \frac{12}{27} & \frac{8}{27} \end{array}$$