Problem solving with circular motion (A2)

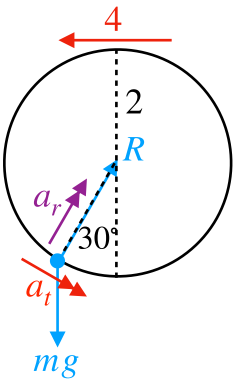
Starter

1. (Review of last lesson) A bead of mass m kg is threaded on a smooth circular wire, of radius 2 m, fixed in a vertical plane. Starting from the highest position the bead has an initial speed of 4 m/s. Point P is when the bead is $30°$ short of reaching the downward vertical. Find as it passes through P:
   (a) the speed of the bead
   (b) the radial acceleration
   (c) the tangential acceleration.

Working: (a) New KE = Initial KE + Loss in GPE

$$\frac{1}{2}mv^2 = \frac{1}{2}m \times 4^2 + mg(2 + 2\cos 30)$$
$$v^2 = 4^2 + 4g(1 + \cos 30)$$
$$v \approx 9.44$$

The speed of the bead is 9.44 m/s (3 s.f.)



(b) $a_r = \dfrac{v^2}{r}$: $\quad a_r = \dfrac{4^2 + 4g(1 + \cos 30)}{2}$

$$a_r \approx 44.6$$

The radial acceleration is 44.6 m/s² (3 s.f.).

(c) Using $F = ma$ tangentially: $\quad mg \sin 60 = ma_t$

$$a_t = g \sin 60 \approx 8.49$$

The tangential acceleration is 8.49 m/s² (3 s.f.).

E.g. 1 A particle P, of mass m kg, moves round a vertical circle of radius 0.5 m with an angular speed of 4 rad/s. Determine whether P is still moving around in a circle $60°$ after the highest point if P is sliding:
   (a) on the inside
   (b) on the outside of a smooth surface.

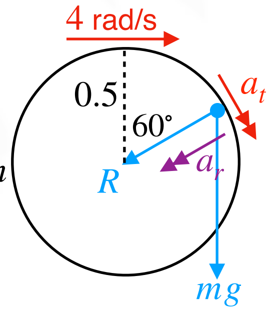
Working: (a) The normal force, R, acts towards the centre.

$$a_r = r\omega^2 = 0.5 \times 4^2 = 8$$

$F = ma$ radially: $\quad R + mg\cos 60 = ma_r$

$$R \approx 3.1m$$

Since $m > 0$, $R > 0$, the particle P is still in contact with the surface.



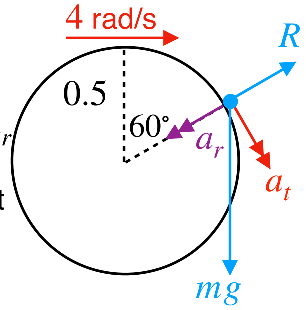
(b) The normal force, R, acts away from the centre.

As above, $a_r = r\omega^2 = 0.5 \times 4^2 = 8$

$F = ma$ radially: $\quad mg\cos 60 - R = ma_r$

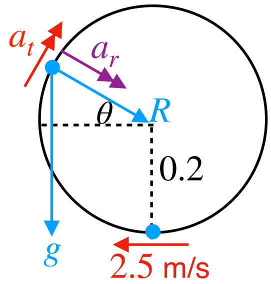
$R = -3.1m$ N

Since $m > 0$, $R < 0$ so the particle P has left the surface.



E.g. 2 A bead P, of mass 1 kg, is threaded on a smooth circular wire, of radius 0.2 m, that is fixed in a vertical plane. P is projected from the lowest point at 2.5 m/s. Find the angle through which OP has rotated when the reaction between the wire and the bead is zero.

Working:



Let θ be the angle past the horizontal through which the bead has turned.

CoE: New KE + Gain in GPE = Initial KE

$$\frac{1}{2} \times 1 \times v^2 + g(0.2 + 0.2\sin\theta) = \frac{1}{2} \times 1 \times 2.5^2$$

$$v^2 = 6.25 - 0.4g(1 + \sin\theta)$$

The normal force, R, acts towards the centre.

$$a_r = \frac{v^2}{r} = \frac{6.25 - 0.4g(1 + \sin\theta)}{0.2}$$
$$= 31.25 - 2g(1 + \sin\theta)$$

$F = ma$ radially: $R + mg\sin\theta = ma_r$

$$R + g\sin\theta = 31.25 - 2g(1 + \sin\theta)$$

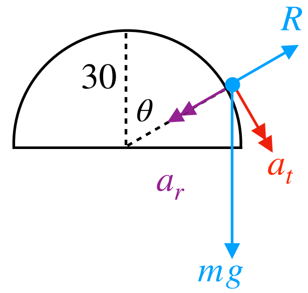
When $R = 0$: $g\sin\theta = 31.25 - 2g(1 + \sin\theta)$

$$3g\sin\theta = 31.25 - 2g$$
$$\sin\theta = \frac{31.25 - 2g}{3g}$$
$$\theta \approx 23.3^\circ$$

The angle through which OP has rotated is $90^\circ + 23.3^\circ = 113^\circ$ (3 s.f.)

E.g. 3 The roof of an arena is a dome in the shape of a hemisphere of radius 30 m. A lump of ice at the top of the dome starts to slide down the surface. Calculate the distance from the base of the dome where the block of ice will hit the ground?

Working: CoE: Gain in KE = Loss in GPE



$$\frac{1}{2}mv^2 = mg(30 - 30\cos\theta)$$
$$v^2 = 60g(1 - \cos\theta)$$
$$a_r = \frac{v^2}{r} = \frac{60g(1-\cos\theta)}{30}$$
$$= 2g(1 - \cos\theta)$$

$F = ma$ radially: $mg\cos\theta - R = ma_r$

$$mg\cos\theta - R = m(2g(1-\cos\theta))$$

The block of ice leaves the surface when $R = 0$:

$$mg\cos\theta = m(2g(1-\cos\theta))$$
$$3g\cos\theta = 2g$$
$$\cos\theta = \frac{2}{3}$$
$$\theta \approx 48.2°$$

At this point the velocity of the ice is given by $v^2 = 60g\left(1 - \frac{2}{3}\right) = 20g$

and its height above the ground is $30\cos\theta = 20$

When the ice leaves the surface of the dome it becomes a projectile with initial velocity $\sqrt{20g}$, angled at about $48.2°$ below the horizontal.

$$\cos\theta = \frac{2}{3} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{5}}{3}$$

Taking ***down*** as ***positive***.

Calculating the time for the ice to hit the ground:

$$s_y = 20,\ a = g,\ u_y = \sqrt{20g}\sin\theta = \frac{\sqrt{100g}}{3},\ t = ?$$

Using $s = ut + \frac{1}{2}at^2$ vertically: $20 = \frac{\sqrt{100g}}{3}t + \frac{1}{2}gt^2$

$$4.9t^2 + \frac{\sqrt{100g}}{3}t - 20 = 0$$

Since $t > 0$, solving gives $t = \frac{10\sqrt{23} - 10\sqrt{5}}{21} \approx 1.2189$ (not -3.3485).

Calculating the horizontal distance travelled after the ice leaves the dome:

$$u_x = \sqrt{20g}\cos\theta = \frac{2}{3}\sqrt{20g},\ t = 1.2189...,\ a = 0,\ s_x = ?$$

Using $s = ut + \frac{1}{2}at^2$ horizontally: $s_x = \frac{2}{3}\sqrt{20g} \times 1.2189$

$$s_x \approx 11.3764$$

Distance from base of the dome is $10\sqrt{+}11.3764 - 30 = 3.74$ m (3 s.f.)

Video: Circular motion
Video: Motion in a vertical circle (sphere)
Video: Circular motion (vertical example)

Exercise

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