

## Product and Addition Principles (single)

### Starter

1. **(Review of last lesson)** The equation  $x^3 + 2x^2 + 3x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . Use the substitution method to find equations which have the following roots.

(a)  $2 + \alpha$ ,  $2 + \beta$  and  $2 + \gamma$                       (b)  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$

**Working:** (a) Let  $u = 2 + \alpha$  then  $\alpha = u - 2$ .  
 Substitute:  $(u - 2)^3 + 2(u - 2)^2 + 3(u - 2) + 4 = 0$   
 Expand and simplify:  $u^3 - 4u^2 + 7u - 2 = 0$   
 If  $u = 2 + \beta$  or  $u = 2 + \gamma$  the same result would be found so  
 $u^3 - 4u^2 + 7u - 2 = 0$  is the required equation.

(b) Let  $u = \alpha^2$  then  $\alpha = \pm \sqrt{u}$   
 $(\pm \sqrt{u})^3 + 2(\pm \sqrt{u})^2 + 3(\pm \sqrt{u}) + 4 = 0$   
 $\pm u\sqrt{u} + 2u \pm 3\sqrt{u} + 4 = 0$   
 $\pm \sqrt{u}(u + 3) = -2u - 4$   
 Squaring both sides:  $u(u^2 + 6u + 9) = 4u^2 + 16u + 16$   
 Expand and simplify:  $u^3 + 2u^2 - 7u - 16 = 0$   
 If  $u = \beta^2$  or  $u = \gamma^2$  the same result would be found so  
 $u^3 + 2u^2 - 7u - 16 = 0$  is the required equation.

2. **(Review of GCSE material)**

A restaurant offers courses including 4 starters, 8 main dishes and 5 desserts.

- (a) Sarah will choose one dish from each category. How many times could she visit the restaurant without having the same meal?  
 (b) Liam will choose one main dish and a dessert. Ryan will choose a starter and a main dish. Who has more choices and by how many?  
 (c) Olivia will choose a main dish and a starter or a dessert. How many options does she have?

**Working:** (a)  $4 \times 8 \times 5 = 120$  days  
 (b) Liam:  $8 \times 5 = 40$  choices,  
 Ryan  $4 \times 8 = 32$  choices  
 so Liam has 8 more options  
 (c)  $32 + 40 = 72$  options

**E.g. 1** Let  $n(A) = 5$ ,  $n(B) = 4$  and  $n(C) = 7$  with  $A$ ,  $B$  and  $C$  being mutually exclusive.

Calculate how many ways there are of doing:

- (a)  $A$ ,  $B$  and  $C$
- (b)  $A$ ,  $B$  or  $C$
- (c)  $A$  and  $C$  only
- (d)  $A$  and ( $B$  or  $C$ )
- (e)  $B$  or ( $A$  and  $C$ )

**Working:** (a)  $5 \times 4 \times 7 = 140$

(b)  $5 + 4 + 7 = 16$

(c)  $5 \times 7 = 35$

(d)  $5 \times (4 + 7) = 55$

(e)  $4 + (5 \times 7) = 39$

**E.g. 2** An examination paper has five questions in section A and three questions section B. How many different ways are there is choose questions if you must choose:

- (a) one question from each section
- (b) a question from either section A or section B.

**Working:** (a)  $5 \times 3 = 15$

(b)  $5 + 3 = 8$

[Solutions to Starter and E.g.s](#)

### Exercise

p3 1A Qu 1i, 2-11, (12 red)