The Exclusion Principle

Starter

1. (Review of last lesson) A team of 5 people is chosen from 8 men and 7 women. How many different teams can be selected if the team must contain:
   (a) 3 men and 2 women
   (b) at least 3 men

Working: (a) Choosing 3 men from 8: $^8C_3 = \dfrac{8!}{5! \times 3!} = 56$

Choosing 2 women from 7: $^7C_2 = \dfrac{7!}{5! \times 2!} = 21$

Total number of teams $= 56 \times 21 = 1176$

(b) The team could have 3, 4 or 5 men in it.

Ways with 3 men: 1176 ways

Ways with 4 men: $^8C_4 \times {}^7C_1 = 70 \times 7 = 490$ ways

Ways with 5 men: $^8C_5 = 56$ ways

So total ways $= 1176 + 490 + 56 = 1722$

2. Describe in words a situation which could be represented by
   (a) $\dfrac{13!}{6!}$ (b) $\dfrac{14!}{9!5!}$ (c) $6!$

Working: (a) Arrange 7 out of 13 different objects

(b) Choose a group of 5 (or 9) from a group of 14 objects

(c) Arrange 6 different objects

3. Four letters are chosen at random from the word RANDOMLY. Find the probability that all 4 letters are consonants.

Working: Total combinations = $^8C_4 = \dfrac{8!}{4!4!} = 70$

Total combinations of 4 consonants = $^6C_4 = \dfrac{6!}{2!4!} = 15$

So probability = $\dfrac{15}{70} = \dfrac{3}{14}$

E.g. 1 The nine members of a committee comprise: one married couple, three more men and four more women.
(a) In how many ways can a working party of five people be selected?
(b) How many of these working parties are such that
   (i) at least one man and at least one woman must be chosen
   (ii) the husband *or* the wife but not both, may be included
   (iii) it is formed entirely of women?

Working (a) $^9C_5 = 126$ ways

(b) (i) Subtract committees with only male members:
$^9C_5 - 1 = 125$ ways

(ii) Subtract committees with both husband and wife:
$^9C_5 - {}^7C_3 = 91$ ways

(iii) 1 way

E.g. 2 Find the number of ways in which a committee of four can be chosen from 6 men and 6 women if:
(a) it must contain 2 men and 2 women
(b) it must contain at least one man
(c) either the youngest man or youngest woman, but not both, must be included?

Working (a) Ways of choosing 2 women from 6: $^6C_2 = 15$
Ways of choosing 2 men from 6: $^6C_2 = 15$
Total ways $= (^6C_2)^2 = 225$ ways

(b) Total number of committees: $^{12}C_4 = 495$
Total committees with no men i.e. all women: $^6C_4 = 15$
Subtract committees of no men from total: $^{12}C_4 - {}^6C_4 = 480$ ways

(c) Ways of including both (choose remaining 2 from 10): $^{10}C_2$
Ways of including neither (choose all 4 from 10): $^{10}C_4$
Ways of including either = Total — both included — neither included
$= {}^{12}C_4 - {}^{10}C_2 - {}^{10}C_4 = 240$

E.g. 3 To inspect a box of 12 light bulbs, a manufacturer selects 4 light bulbs from the box and rejects the box if more than 1 bulb is faulty. If a box has 3 faulty light bulbs, find the probability that the box will be accepted.

Working Total ways of choosing = $^{12}C_4 = 495$ ways
There are 9 non-faulty light bulbs.
To select the box at least 3 of the light bulbs must not be faulty.
Using the exclusion principle
3 faulty, 1 non-faulty = $1 \times 9 = 9$ ways
2 faulty, 2 non-faulty = $^3C_2 \times {}^9C_2 = 3 \times 36 = 108$ ways
So probability the box will be accepted $= \dfrac{495 - 9 - 108}{495}$
$= \dfrac{378}{495} = \dfrac{42}{55} \approx 0.76$

Not using the exclusion principle
4 non-faulty = $^9C_4 = 126$ ways
3 non-faulty, 1 faulty = $^9C_3 \times {}^3C_1 = 84 \times 3 = 252$ ways
So probability the box will be accepted $= \dfrac{126 + 252}{495}$
$= \dfrac{378}{495} = \dfrac{42}{55} \approx 0.76$

Exercise
p11 1D Qu 1-7, (8-9 red)