

Transformations of Discrete Random Variables

Starter

1. **(Review of last lesson)** A committee of three is to be chosen from 7 girls and 4 boys. Find the expected number of boys on the committee.

Working: *We could choose 0, 1, 2 or 3 boys.*

No boys: 7C_3 ways
 1 boys: ${}^4C_1 \times {}^7C_2$ ways
 Total committees: ${}^{11}C_3$ ways
 The full table looks like this:

$b :$	0	1	2	3
$P(X = x) :$	$\frac{{}^7C_3}{{}^{11}C_3}$	$\frac{{}^4C_1 \times {}^7C_2}{{}^{11}C_3}$	$\frac{{}^4C_2 \times {}^7C_1}{{}^{11}C_3}$	$\frac{{}^4C_3}{{}^{11}C_3}$
$P(X = x) :$	$\frac{7}{33}$	$\frac{28}{55}$	$\frac{14}{55}$	$\frac{4}{165}$

$$E(\text{boys}) = \frac{28}{55} + 2 \times \frac{14}{55} + 3 \times \frac{4}{165}$$

$$= \frac{12}{11}$$

2. A random variable, X , has distribution:

$$P(X = x) : \begin{array}{ccc} x : & 1 & 2 & 3 \\ & 0.2 & 0.3 & 0.5 \end{array}$$

Give all your answers exactly.

- (a) Find the mean of X , i.e. $E(X)$, $\text{Var}(X)$ and the standard deviation.
- (b) Let each x -value be multiplied by 3, leaving the probabilities the same. Find the expectation, variance and standard deviation of $3X$.
- (c) Let each original value of x be increased by 4, leaving the probabilities the same. Find the expectation, variance and standard deviation of $X + 4$.
- (d) Explain your answers to (b) and (c) with regards to the standard deviation of data.

Working: (a) $E(X) = 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.5 = 2.3$
 $E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.3 + 3^2 \times 0.5 = 5.9$
 $\text{Var}(X) = 5.9 - 2.3^2 = 0.61 = \frac{61}{100}$

Standard deviation, $\sigma = \frac{\sqrt{61}}{10} \approx 0.781$ (3 s.f.)

(b) $E(3X) = 3 \times 0.2 + 6 \times 0.3 + 9 \times 0.5 = 6.9$
 $E((3X)^2) = 3^2 \times 0.2 + 6^2 \times 0.3 + 9^2 \times 0.5 = 53.1$
 $\text{Var}(3X) = 53.1 - 6.9^2 = 5.49 = \frac{549}{100}$

Standard deviation, $\sigma = \frac{3\sqrt{61}}{10} \approx 2.34$ (3 s.f.)

$$\begin{aligned} \text{(c)} \quad E(X + 4) &= 5 \times 0.2 + 6 \times 0.3 + 7 \times 0.5 = 6.3 \\ E((X + 4)^2) &= 5^2 \times 0.2 + 7^2 \times 0.3 + 9^2 \times 0.5 = 40.3 \\ \text{Var}(X + 4) &= 40.3 - 6.3^2 = 0.61 = \frac{61}{100} \\ \text{Standard deviation, } \sigma &= \frac{\sqrt{61}}{10} \approx 0.781 \text{ (3 s.f.)} \end{aligned}$$

(d) Multiplying the values by a scalar changes the spread, but adding does not.

E.g. 1 The discrete random variable X that has p.d.f.:

$x :$	2	6	9
$P(X = x) :$	0.4	0.25	0.35

Find:

- | | |
|-----------------------------------|-------------------|
| (a) the mean of X , i.e. $E(X)$ | (b) $E(X^2)$ |
| (c) $E(3X - 1)$ | (d) $E(2X^2 + 4)$ |

Working:

$$\begin{aligned} \text{(a)} \quad E(X) &= 2 \times 0.4 + 6 \times 0.25 + 9 \times 0.35 = 5.45 \\ \text{(b)} \quad E(X^2) &= 2^2 \times 0.4 + 6^2 \times 0.25 + 9^2 \times 0.35 = 38.95 \\ \text{(c)} \quad E(3X - 1) &= 3E(X) - 1 = 3 \times 5.45 - 1 = 15.35 \\ \text{(d)} \quad E(2X^2 + 4) &= 2E(X^2) + 4 = 2 \times 38.95 + 4 = 81.9 \end{aligned}$$

E.g. 2 Find the exact value of $\text{Var}(2X + 3)$ for the discrete random variable X that has probability distribution:

$x :$	1	2	3	4
$P(X = x) :$	0.375	0.125	0.25	0.25

Working:

$$\begin{aligned} E(X) &= \frac{19}{8} = 2.375 \\ E(X^2) &= \frac{57}{8} = 7.125 \\ \text{Var}(X) &= \frac{57}{8} - \left(\frac{19}{8}\right)^2 = \frac{95}{64} = 1.484375 \\ \text{Var}(2X + 3) &= 2^2 \times \text{Var}(X) = 4 \times \frac{95}{64} = \frac{95}{16} = 5.9375 \end{aligned}$$

Video: [Linear combinations of discrete random variables](#)

[Discrete random variable EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p27 2B Qu 1i, 2-6, (7 red)