

## Work done by a force (single)

### Starter

1. Prove that  $2n + 1 < 2^n$  for all integers  $n > 3$ .

**Working:** *(Proposition)*

Let  $P(n)$  be the proposition that  $2n + 1 < 2^n$  for  $n > 3$ .

*(Prove the basic case)*

When  $n = 4$ :  $(2 \times 4 + 1) = 9 < 2^4 = 16$  which is true.

Therefore  $P(4)$  is true.

*(Inductive step)*

Assume that  $P(k)$  is true i.e.  $2k + 1 < 2^k$  for  $k > 3$ .

*(Inductive step – consider the next term)*

Need to prove  $P(k + 1)$  is true i.e.  $2(k + 1) + 1 < 2^{k+1}$

*(Inductive step – algebraic manipulation)*

$$\begin{aligned}
 P(k + 1): \quad 2^{k+1} &= 2 \times 2^k \\
 &> 2 \times (2k + 1) && \text{assuming } P(k) \text{ is true} \\
 &= 4k + 2 \\
 &= 2k + 2k + 2 \\
 &> 2k + 3 && \text{since } 2k + 2 > 3 \text{ for } k > 3 \\
 &= 2(k + 1) + 1
 \end{aligned}$$

*(Completion)*

But this is  $P(k)$  with  $k$  replaced by  $k + 1$ .

Therefore, if  $P(k)$  is true, then  $P(k + 1)$  is also true.

$P(1)$  is true and if  $P(k)$  is true, then  $P(k + 1)$  is true. Using the principle of mathematical induction,  $P(n)$  is true for all positive integers.

- E.g. 1** A car of mass 1200 kg travels 5 km at a constant speed along a road against resistive force of constant friction and air resistance force of 800 N and 500 N respectively. Calculate the work done by the engine.

**Working:** Since the car moves at a constant speed the work done by the engine equals the work done against the resistive forces  
 Work done =  $(800 + 500) \times 5000 = 6500 \text{ kJ}$

- E.g. 2** A gardener moves a wheelbarrow 30 metres along a level, straight path. The work done by the gardener is 120 J, and the barrow is initially and finally at rest. Calculate the average force resisting the motion.

**Working:**  $WD = Fs$ :  $120 = F \times 30 \Rightarrow F = 4$   
 The average force resisting the motion is 4 N.

- E.g. 3** A weightlifter raises a mass of 120 kg from the ground to above her head – a distance of 2 m. Calculate the work done against gravity. Assume  $g = 9.8$ . Give your answer to the nearest J.

**Working:**  $WD = 120 \times 9.8 \times 2 = 2352 \text{ J}$

**E.g. 4** A man of mass 85 kg climbs 2 m up a ladder which is inclined at  $70^\circ$  to the horizontal. Calculate the work done against gravity. Give your answer to the nearest J.

**Working:**  $WD = 85g \times 2 \sin 70 = 1566 \text{ J}$

**Video:** [Work done by a force \(horizontal plane\)](#)  
**Video:** [Work done by a force \(vertical plane\)](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p5 1A Qu 1-8

