Work done by a force acting at an angle to the direction of motion

Starter

1. If no mechanical energy were lost, a skier descending a straight 40 m slope would arrive at the foot with speed 15 m/s. Calculate the angle the slope makes with the horizontal given that the skier set off from rest.

Working: Let the angle of the slope be θ.

Gain in KE = Loss in GPE: $\frac{1}{2} \times m \times 15^2 = mg \times 40 \sin\theta$

$$\sin\theta = \frac{225}{80g}$$

$$\theta \approx 16.7$$

The angle the slope makes with the horizontal $16.7°$

2. A cyclist climbs a hill of length 3 km which has an average gradient of $6\,\%$. At the bottom of the hill the cyclist is travelling at 30 km/h but at the top his speed has dropped to 12 km/h. Given that the cyclist's average driving force is 250 N and the combined mass of bike and rider is 80 kg, calculate the average resistive force.

Working: Let the average resistive forces be F.

WD by driving forces + Initial ME = WD against resistive forces + Final ME

$$250 \times 3000 + \frac{1}{2} \times 80 \times \left(\frac{25}{3}\right)^2 = 3000F + \frac{1}{2} \times 80 \times \left(\frac{10}{3}\right)^2 + 80g \times 3000 \sin 6$$

$F \approx 168.8$ N

the average resistive force is 169 N

3. A force of 12 N acts at an angle of $30°$ to the horizontal. How much of this force acts:
(a) in the horizontal direction (b) in the vertical direction.

Working: (a) $12\cos 30 = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} = 10.4$ N

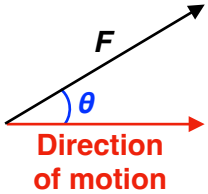
(b) $12\sin 30 = 12 \times \frac{1}{2} = 6$ N

4. A force of R N acts at an angle of θ to the horizontal. How much of this force acts:
(a) in the horizontal direction (b) in the vertical direction.

Working: (a) Horizontal $= R\cos\theta$

(b) Vertical $= R\sin\theta$

E.g. 1 Work done = Force × Distance when the force is acting in the direction of motion i.e. $WD = Fs$. What is the work done when the force, F, is acting at an angle of θ to the direction of motion?



Working: $WD = F\cos\theta \times$ distance

E.g. 2 A sledge is pulled along level ground by a rope inclined at $20°$ to the horizontal. Given that the tension in the rope is 30 N, calculate the work done, to the nearest J, in moving the sledge 40 m.

Working: Work done $= 30\cos 20 \times 40 = 1128$

Video: Work done on rough inclined planes

Solutions to Starter and E.g.s

Exercise
p16 1D Qu 1-10

Summary
When the force, whether driving or resistive, acts at an angle to the direction of motion, ***resolve the force in the direction of motion*** before multiplying by the distance.