

L6 Mathematics
January Exam
Paper 1 (Teacher X)
January 2025
2024-2025

Duration: 1 hour 5 minutes

Total number of marks: 54

Write your answers on file paper provided.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is given below:

Formulae

AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots + {}^n C_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1.

Express $\frac{\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{3}}$ in the form $a + \sqrt{b}$, where a and b are integers.

Fully justify your answer.

[4 marks]

2.

Use logarithms to solve the equation

$$5^{x-2} = 71570$$

Give your answer to two decimal places.

[3 marks]

3

Points A and B have position vectors $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = -\mathbf{i} + 5\mathbf{j}$, respectively, where O is the origin.

(a) Find the position vector of the midpoint of AB . [1]

(b) Find the exact magnitude of the vector \overrightarrow{AB} . [3]

(c) Find the angle between the vector \overrightarrow{OA} and the positive x -axis. [2]

4.

It is given that

$$3 \log_a x = \log_a 72 - 2 \log_a 3$$

Solve the equation to find the value of x

Fully justify your answer.

[4 marks]

5.

The quadratic equation $kx^2 + 2kx + 2k = 3x - 1$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$4k^2 + 16k - 9 > 0. \quad [4]$$

(b) Hence find the set of possible values of k . Give your answer in set notation. [2]

6.

Differentiate the following with respect to x .

(a) $3x^4 - \frac{2}{x^2}$ [2]

(b) $4\sqrt{x} - 9$ [2]

7.

It is given that

$$\ln x - \ln y = 3$$

(a) Express x in terms of y in a form not involving logarithms. [3 marks]

(b) Given also that

$$x + y = 10$$

find the exact value of y and the exact value of x

[3 marks]

8.

Express $\frac{a^{\frac{7}{2}} - a^{\frac{5}{2}}}{a^{\frac{3}{2}} - a}$ in the form $a^m + \sqrt{a^n}$, where m and n are integers and $a \neq 0$ or 1 . [5]

9.

At the point P on the curve $y = e^{3x} - 21x - 8$ the gradient of the tangent is 3.

(a) Determine the x -coordinate of P . Give your answer in the form $\ln a$, where a is an integer to be determined. [4]

(b) Hence determine the equation of the tangent at P . Give your answer in the form $y = 3x + c$ where c is an exact constant to be determined. [3]

10.

The world human population, P billions, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after 2004

Using the estimated population figures for the years from 2004 to 2007, a graph is plotted of $\log_{10}P$ against t .

The points lie approximately on a straight line with

- gradient 0.0054
- intercept 0.81 on the $\log_{10}P$ axis

(a) Estimate, to 3 decimal places, the value of a and the value of b .

(4)

In the context of the model,

(b) (i) interpret the value of the constant a ,

(ii) interpret the value of the constant b .

(2)

(c) Use the model to estimate the world human population in 2030

(2)

(d) Comment on the reliability of the answer to part (c).

(1)