

L6 Ma January Exam Teacher Y 24-25 SOLUTIONS [53]

1.

2(a)	$(f(4) =) 2 \times 4^3 - 3a \times 4^2 + 4b + 8a = 0$	M1	1.1b
	$128 + 4b = 40a \Rightarrow 32 + b = 10a *$	A1*	1.1b
		(2)	
(b)	$f(2) = 2 \times 2^3 - 3a \times 2^2 + 2b + 8a = 0 \Rightarrow 8 + b = 2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a = \dots$ or $\Rightarrow b = \dots$	dM1	2.1
	$a = 3$ or $b = -2$ or $k = 3$	A1	1.1b
	$(f(x) =) (2x + 3)(x - 4)(x - 2)$	A1	1.1b
		(4)	
(c)(i) (ii)	3	B1	1.1b
	12	B1ft	2.2a
		(2)	
(8 marks)			

2.

(a)	$3[(x + 4)^2 - 16] + 5 \dots$	$a = 3$	B1	1.1	No marks until attempt to complete the square Must be of the form $3(x \pm \alpha)^2 \pm \dots$ where $\alpha \neq 0$	If they compare coefficients $a = 3$ B1 , $b = 4$ B1 , $c = -43$ B1 $3(x^2 + 4)^2 - 43$ scores B0 B0 B1
		$(x + 4)^2$	B1	1.1	Allow $(x + \frac{8}{2})^2$	
	$3(x + 4)^2 - 43$	$c = -43$	B1	1.1	If expression set to 0, ignore = 0	
					[3]	
(b)	$y \leq 0$ $y \geq 3x^2 + 24x + 5$		B1 B1	1.1 1.1	Accept $-43 \leq y \leq 0$ If B0 B0 then SC B1 if both 'correct' but with strict inequalities	
					[2]	

3.

(a)	DR $(x + 2)^4 - 6(x + 2)^2 - 16$ $x^4 + 8x^3 + 24x^2 + 32x + 16 - 6(x^2 + 4x + 4) - 16$ $= x^4 + 8x^3 + 18x^2 + 8x - 24$ AG	B1	1.1	For this correct intermediate step or an equivalent correct expression after expansions leading to AG or a clear conclusion www.
				[1]
(b)	DR $(x + 2)^4 - 6(x + 2)^2 - 16 = 0$ $((x + 2)^2 - 8)((x + 2)^2 + 2) = 0$ or $(x + 2)^2 = \frac{6 \pm \sqrt{36 + 64}}{2}$ oe $(x + 2)^2 = 8$ or -2 $(x + 2)^2 = -2$ has no roots $x = -2 + \sqrt{8}$ or $-2 - \sqrt{8}$	M1 A1 A1 A1	3.1a 2.1 2.3 1.1	(Applying the given identity) or $(y^2 - 8)(y^2 + 2) = 0$ or $(z - 8)(z + 2) = 0$ or $y^2 = \frac{6 \pm \sqrt{36 + 64}}{2}$ Attempt to factorise or apply formula or $y^2 = 8$ or -2 soi Somehow indicated or $y^2 = -2$ has no roots or $x = -2 + 2\sqrt{2}$ or $-2 - 2\sqrt{2}$ or $x = 0.828$ or -4.83 (3 sf) Answers only receive no credit (DR). SC B1B1 (max 2/4) for correct answers and 'no roots' case ($x = -2$) if only the working to solve the first quadratic is omitted.
				[4]

4.

4(a)	$\frac{x}{\sin 30^\circ} = \frac{x+3}{\sin 140^\circ}$	M1	1.1b
	$2x \sin 140 - x = 3$	dM1	3.1a
	$x = 10.5 \quad *$	A1*	1.1b
		(3)	
(b)	$AB^2 = 8.5^2 + 13.5^2 - 2 \times 8.5 \times 13.5 \times \cos 150^\circ$ $\Rightarrow AB = \dots$	M1	3.1a
	$AB = \text{awrt } 21.3 \text{ (cm)}$	A1	1.1b
		(2)	
(5 marks)			

5.

$[(x-3)^2 - 9 + y^2 = 16 \text{ or } (x-3)^2 + y^2 = 25]$ Centre (3, 0) radius 5 Translation of $\begin{pmatrix} k \\ k \end{pmatrix}$ soi $k = 2 \times 5 \cos 45^\circ$ Centre of D is $(3 + 5\sqrt{2}, 5\sqrt{2})$	B1	2.1	May not be seen
	B1	1.1	soi
	M1	1.1	e.g. $(3 + \alpha, \alpha)$
	M1	1.1	For use of the distance between centres C and D being $2r$ (look for attempts at Pythagoras in an isosceles triangle with hypotenuse 10) e.g. $2 \times \frac{5}{\sqrt{2}}$ or $5\sqrt{2}$ or $\frac{10}{\sqrt{2}}$ oe or 7.07 or 7.1
	A1	2.2a	(10.1, 7.1)
	[5]		

6.

(a)	$\left(5, \frac{8+p}{2}\right)$	B1	1.1	Allow unsimplified e.g. $\left(\frac{4+6}{2}, \frac{8+p}{2}\right)$ Allow $x = \frac{4+6}{2}, y = \frac{8+p}{2}$	ISW if correct unsimplified answer is incorrectly simplified
		[1]			
(b)	$(m_{AB}) = \frac{p-8}{2}$	B1	1.1	Allow unsimplified e.g. $\frac{8-p}{4-6}$	ISW if correct unsimplified answer is incorrectly simplified
		[1]			
(c)	$(m_{\perp}) = \frac{2}{8-p}$ $y - \left(\frac{8+p}{2}\right) = \frac{2}{8-p}(x-5)$ $0 - \left(\frac{8+p}{2}\right) = \frac{2}{8-p}\left(\frac{7}{5}p - 5\right)$ $5p^2 - 28p - 220 = 0$ or $5p^3 - 68p^2 + 4p + 1760 = 0$ $(p = 10 \text{ and}) \text{ so } x = 14$	B1FT	2.1	Correctly applies $m_1 m_2 = -1$ to their answer to part (b) to obtain the gradient of the line perp. to AB	Using their part (a) or $\left(\frac{7}{5}p, 0\right)$ and their perp. gradient to AB Or their part (a)
		M1*	3.1a	Setting up an equation in p for the line through the midpoint of AB perp. to AB. Could use $y = mx + c$ and obtain c in terms of p .	
		M1dep*	1.1	Substituting $\left(\frac{7}{5}p, 0\right)$ into their line. Could obtain another expression for c using the other point and equate the two expressions of c .	
		A1	1.1	Expand and simplify to a 3TQ in p or possibly a 4 term cubic in p .	
		A1	3.2a	BC cao	
		[5]			

7.

(a)	$y = \sqrt{8x-1}$	M1 A1 [2]	1.1 1.2	x in $\sqrt{2x-1}$ replaced with either $4x$ or $\frac{1}{4}x$ cao	Must have $y = \dots$
(b)	(1, 3)	B1 B1 [2]	3.1a 1.1	Correct x -coordinate Correct y -coordinate Accept $x=1, y=3$	(1, ± 3) scores B1B0 unless -3 is rejected. (1/3) scores SC B1

8.

6(a)	e.g. ${}^{12}C_1 \times a = -\frac{15}{2}$	M1	1.1b
	$12a = -\frac{15}{2} \Rightarrow a = -\frac{5}{8}$ *	A1*	1.1b
		(2)	
(b)	${}^{12}C_2 \times a^2 = k \Rightarrow k = \dots$	M1	1.1b
	$k = \frac{825}{32}$	A1	1.1b
		(2)	
(c)	$1 - \frac{5}{8}x = \frac{17}{16} \Rightarrow x = -0.1 \Rightarrow 1 - \frac{15}{2} \times (-0.1) + \frac{825}{32} \times (-0.1)^2$	M1	3.1a
	= awrt 2.0078	A1	1.1b
		(2)	

(6 marks)

9.

10(a)	tangent = -2 \rightarrow normal = $\frac{1}{2}$	B1	2.2a
	$\frac{k^2 - 2k - k - 8}{3 - -1}$	M1	1.1b
	$\frac{1}{2} = \frac{k^2 - 2k - k - 8}{3 - -1} \Rightarrow \frac{1}{2} = \frac{k^2}{4} - \frac{2k}{4} - \frac{k}{4} - 2$	dM1	1.1b
	$\Rightarrow k^2 - 3k - 10 = 0$ *	A1*	2.1
		(4)	
(b)	$k = 5$	B1	2.3
	y coordinate of P is 13	B1ft	1.1b
	Attempts $PQ \left(= \sqrt{(3 - -1)^2 + (25 - 15 - 8)^2} \right)$ (or PQ^2)	M1	3.1a
	$(x+1)^2 + (y-13)^2 = 20$	A1	1.1b
		(4)	

(8 marks)