L6 Mock Teacher X 19-20 SOLUTIONS [73]

1.

$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
Deduces that $a = 0$ is a solution		B1	2.2a
		(4)	
$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2 + 9)(b^2 - 2) = 0$		M1	1.1b
$b^2 = -9, 2$		A1	1.1b
$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
		(4)	
			(8 marks)

2.

(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

3.

Multiplies by $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$	AO1.1a	M1	$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
Correctly evaluates denominator to get 3 – 2 or 1	AO1.1b	A1	$= \frac{\sqrt{18}+\sqrt{12}}{3-2}$
Evaluates numerator, one term correct $\sqrt{18}$ *or* $\sqrt{12}$ or $3\sqrt{2}$ *or* $2\sqrt{3}$	AO1.1b	A1	$\frac{\sqrt{18}+\sqrt{12}}{1}$
Completes solution CAO	AO2.1	R1	$= \sqrt{9 \times 2} + \sqrt{4 \times 3}$ $= 3\sqrt{2} + 2\sqrt{3}$
Total		**4**	

4.

Uses a law of logarithms correctly in their working from the list below: Multiplication / Division / Power **NB Any attempt to show the result with numerical values scores 0/4**	1.1a	M1	$\log_{10}\frac{x^4}{100}+\log_{10}9x-\log_{10}x^3$ $=4\log_{10}x-\log_{10}100+\log_{10}9+\log_{10}x-3\log_{10}x$ $=-2\log_{10}10+2\log_{10}3+2\log_{10}x$ $=2(-\log_{10}10+\log_{10}3+\log_{10}x)$ $=2(-1+\log_{10}3x)$
Uses a different law of logarithms correctly from above list NB $\log_{10}\frac{9x^2}{100}$ OE scores **M1 M1**	1.1a	M1	
Obtains at least two terms equivalent to $-2\log_{10}10+2\log_{10}3+2\log_{10}x$	1.1b	A1	
Completes rigorous argument with no slips to obtain $2(-1+\log_{10}3x)$ correctly with Base 10 identified in the final answer AG	2.1	R1	
Total		**4**	

5.

5	**(i)**		$(2^{-2})^3$ or $2^{15}\div 2^{21}$ 2^{-6}	**B1** **B1** **[2]**	Valid attempt to simplify Correct answer. Accept $p=-6$.	Correct use of either index law $\left(\frac{1}{2}\right)^6$ **oe** is **B1**
5	**(ii)**		$5\times(2^2)^{\frac{2}{3}}+3\times(2^4)^{\frac{1}{3}}$ $=5\times2^{\frac{4}{3}}+3\times2^{\frac{4}{3}}$ or $10\times2^{\frac{1}{3}}+6\times2^{\frac{1}{3}}$ $=8\times2^{\frac{4}{3}}$ $=2^{\frac{13}{3}}$	**M1** **B1** **A1** **[3]**	Attempts to express both terms or a combined term as a power of 2 Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for **either** term Correct final answer	e.g. **Both** $4=2^2$ **and** $16=2^4$ **soi** If **M0** **SC B1** for $8\times16^{\frac{1}{3}}$ or $8\times4^{\frac{2}{3}}$

6.

(a)	$x^n\to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right)=6x-\frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
b)	Attempts $6x-\frac{24}{x^2}>0\Rightarrow x>$	M1	1.1b
	$x>\sqrt[3]{4}$ or $x\geqslant\sqrt[3]{4}$	A1	2.5
		(2)	

(5 marks)

7.

6	**(i)**		$-2(x^2-6x-2)$ $=-2[(x-3)^2-2-9)]$ $=-2(x-3)^2+22$	**B1** **B1** **M1** **A1** **[4]**	or $a=-2$ $b=-3$ $4+2b^2$ $c=22$ If a, b and c found correctly, then **ISW** slips in format. **If signs of all terms changed at start, can only score SC B1** for fully correct working to obtain $2(x-3)^2-22$ If done correctly and then signs changed at end, do not **ISW**, award **B1B1M1A0**	$-2(x-3)^2-22$ **B1 B1 M0 A0** $-2(x-3)+22$ **4/4 (BOD)** $-2(x-3x)^2+22$ **B1 B0 M1 A0** $-2(x^2-3)^2+22$ **B1 B0 M1 A0** $-2(x+3)^2+22$ **B1 B0 M1 A0** $-2x(x-3)^2+22$ **B0 B1 M1 A0** $-2(x^2-3)+22$ **B1 B0 M1 A0**
6	**(ii)**		(3, 22)	**B1ft** **B1ft** **[2]**	Allow follow through "– their b" Allow follow through "their c"	May restart. Follow through marks are for **their final answer** to (i)

8.

8	**(i)**		$y_1=50$, $y_2=2(5+h)^2$ $\frac{(50+20h+2h^2)-50}{(5+h)-5}$ $20+2h$	**B1** **M1** **A1** **[3]**	Finds y coordinates at 5 and $5+h$ Correct method to find gradient of a line segment; at least 3/4 values correct Fully correct working to give answer **AG**	Need not be simplified
8	**(ii)**		e.g. "As h tends to zero, the gradient will be 20"	**B1** **[1]**	Indicates understanding of limit **See Appendix 2 for examples**	e.g. refer to h tending to zero or substitute $h=0$ into $20+2h$ to obtain gradient at A
8	**(iii)**		Gradient of normal $=-\frac{1}{20}$ $y-50=-\frac{1}{20}(x-5), x=0$ 50¼	**B1** **M1** **A1** **[3]**	 Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\frac{201}{4}, \frac{1005}{20}$	 Any correct method e.g. labelled diagram.

9.

4 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt}=-3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T}=-500 \Rightarrow 3925e^{-0.25T}=500$ * cso	A1*	3.4
	(ii) $e^{-0.25T}=0.127... \Rightarrow -0.25T=\ln 0.127...$	M1	1.1b
	$T=8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

10.

Selects differentiation as the first step. At least one term correct	1.1a	M1	$y = 2x^5 + 5x^4 + 10x^3 - 8$ $\frac{dy}{dx} = 10x^4 + 20x^3 + 30x^2$
Differentiates fully correctly	1.1b	A1	
Equates their derivative to zero	1.1a	M1	$10x^2(x^2 + 2x + 3) = 0$
States $x = 0$ is one solution or verifies $x = 0$ is a solution	1.1b	A1	$x = 0$ or $x^2 + 2x + 3 = 0$
Deduces the quadratic factor has no real roots using discriminant, completing the square, using formula Or uses a sketch from their calculator Or finds roots of quartic but discounts non-real roots (only real root is x = 0)	2.2a	M1	discriminant = $b^2 - 4ac$ = 4 – 12 = –8 negative so no real solutions Only stationary point at (0, –8)
Deduces that there are no further stationary points and concludes that (0, –8) is the only one.	2.1	R1	
Total		**6**	

11.

9			$x^2 + (2 - 2k)x + 11 + k = 0$ $(2 - 2k)^2 - 4(11 + k)$ $4k^2 - 12k - 40 > 0$ $k^2 - 3k - 10 > 0$ $(k - 5)(k + 2)$ $k < -2, k > 5$	**M1*** **M1dep*** **A1** **M1dep*** **A1** **M1dep*** **A1** **[7]**	Attempt to rearrange to a three-term quadratic Uses $b^2 - 4ac$, involving k and not involving x Correct simplified inequality obtained **www** Correct method to find roots of 3-term quadratic 5 and – 2 seen as roots $b^2 - 4ac > 0$ and chooses "outside region" Fully correct, strict inequalities.	Each Ms depend on the previous M $-2 > k > 5$ scores **M1A0** Allow "$k < -2$ or $k > 5$" for A1 **Do not allow "$k < -2$ and $k > 5$"**

12.

(a)	Reads graph and uses $10^{\log P}$ or $10^{\log V}$ to get either P or V	AO3.4	M1	$\log_{10} P = 2.18$ $P = 151$
	Correctly obtains P and V AWRT 150 and AWRT 0.71	AO1.1b	A1	$\log_{10} V = -0.15$ $V = 0.708$
b)	Calculates value of gradient to find d Condone use of log d = gradient Or uses a value of c plus a P/V pair to find d	AO3.4	M1	$\log_{10} P = \log_{10} c + d \log_{10} V$
	Obtains correct value for d AWRT –1.4 Not necessarily a decimal	AO1.1b	A1	Gradient = d = –1.4
	Calculates value of intercept to find $\log_{10} c$ Or uses a value of d plus a P/V pair to find c	AO3.4	M1	Intercept = $\log_{10} c$
	Calculates correct value for c AWRT 93	AO1.1b	A1	$c = 93.3$
c)	Uses their values of c and d in the formula $P = cV^d$	AO1.1a	M1	$P = 93.3 \times 2^{-1.4}$
	Obtains P value, **including units** AWFW 30 to 40	AO3.2a	A1	= 35.4 kilopascals
	Total		**8**	